UNIWERSYTET ŁÓDZKI

Cz. 1

Spin, pseudospin, chiralność, krzywizna Berryego czyli od elektroniki do dolinotroniki (valleytroniki). Droga od grafenu do struktur hybrydowych i nie tylko.

FACULTY OF PHYSICS AND APPLIED INFORMATICS
University of Lodz

prof. dr hab.
Zbigniew Klusek

Nowe wyzwania

Koszty budowy fabryki:
kilkanaście miliardów $

Co 18 miesięcy podwaja się liczba tranzystorów

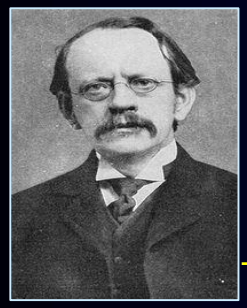
Tranzystory z bramką 16-10 nm w masowej produkcji

Na razie proces skalowania się udaje

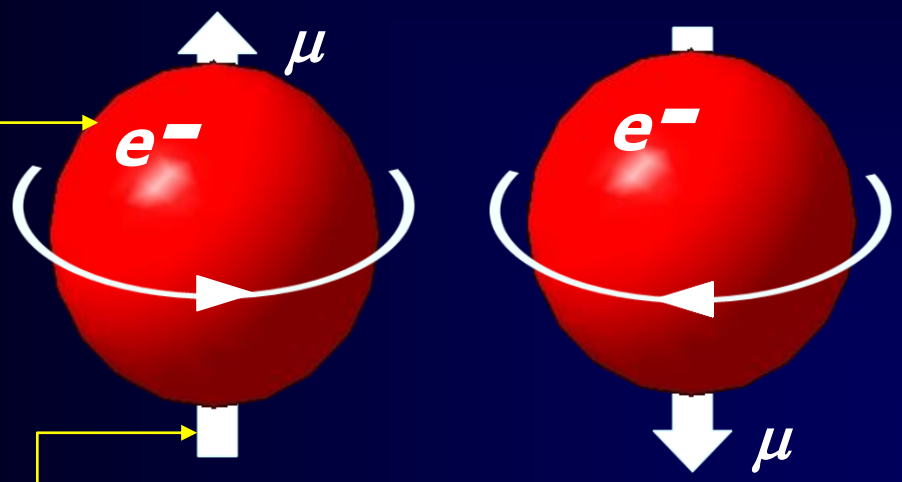
POJAWIA SIĘ MOTYWACJA DO WPROWADZANIA TECHNOLOGII OPARTYCH NA NOWYCH ZJAWISKACH I MATERIAŁACH

Nowe wyzwania

Poszukujemy nowych wewnętrznych kwantowych stopni swobody elektronu

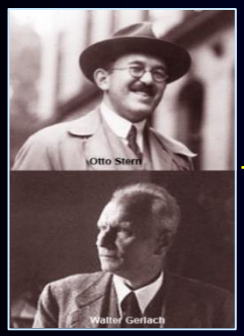


J.J. Thomson



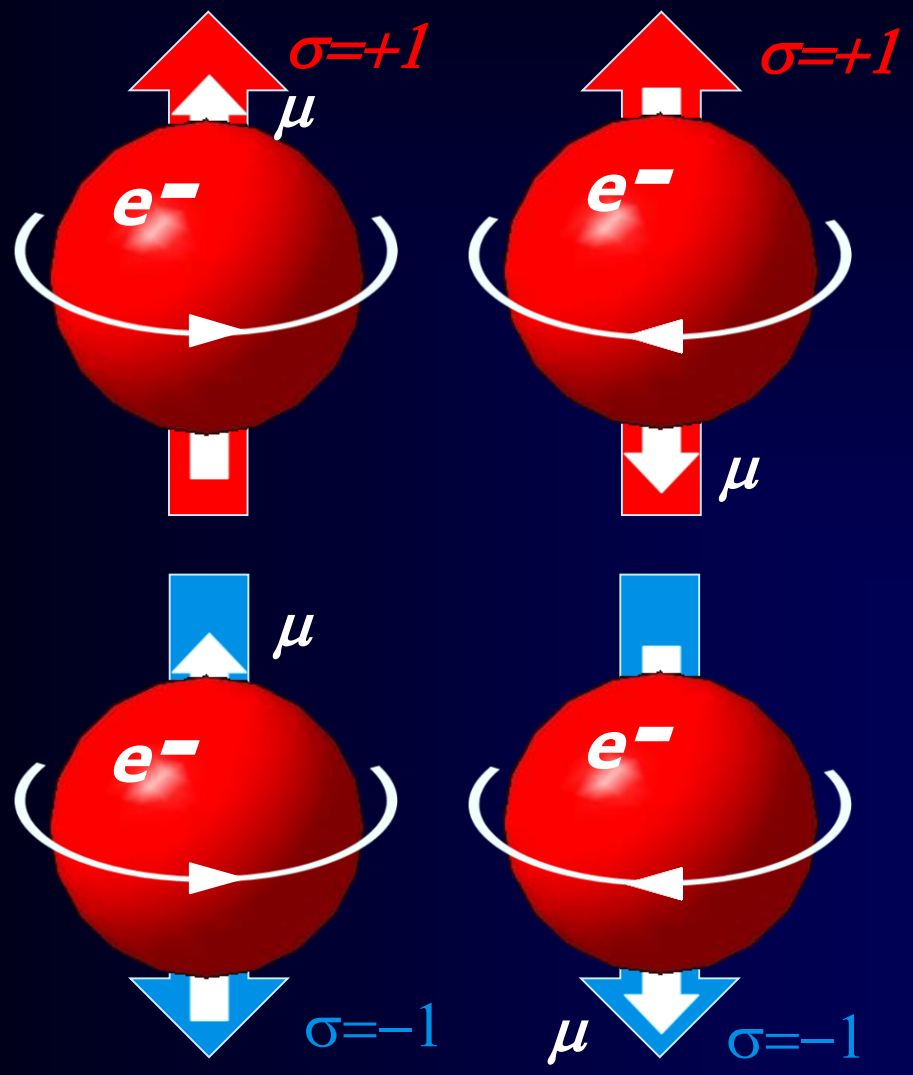
ładunek elektryczny
elektronika

spin
spintronika



O. Stern, W. Gerlach

Nowe możliwości

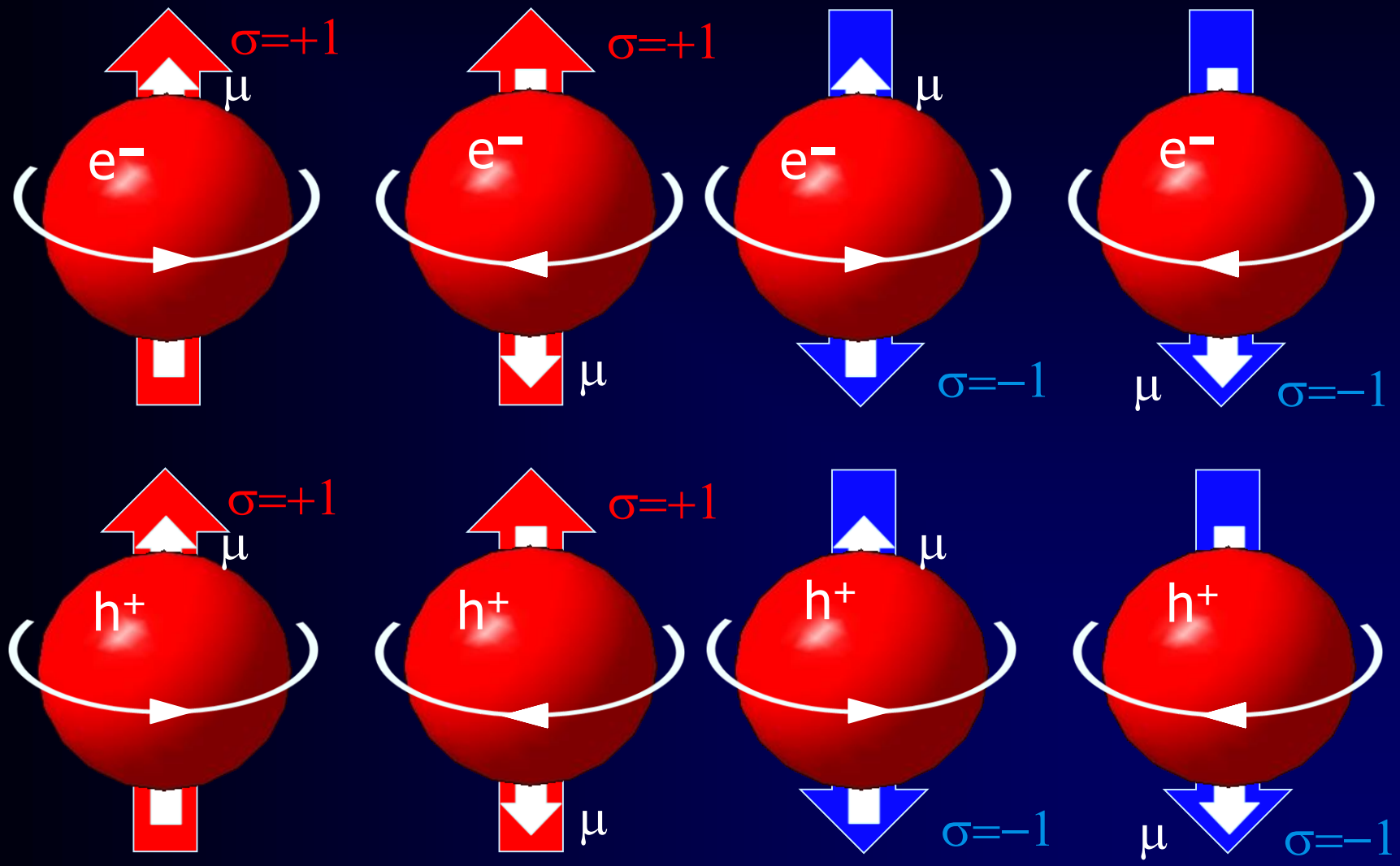


ładunek elektronika

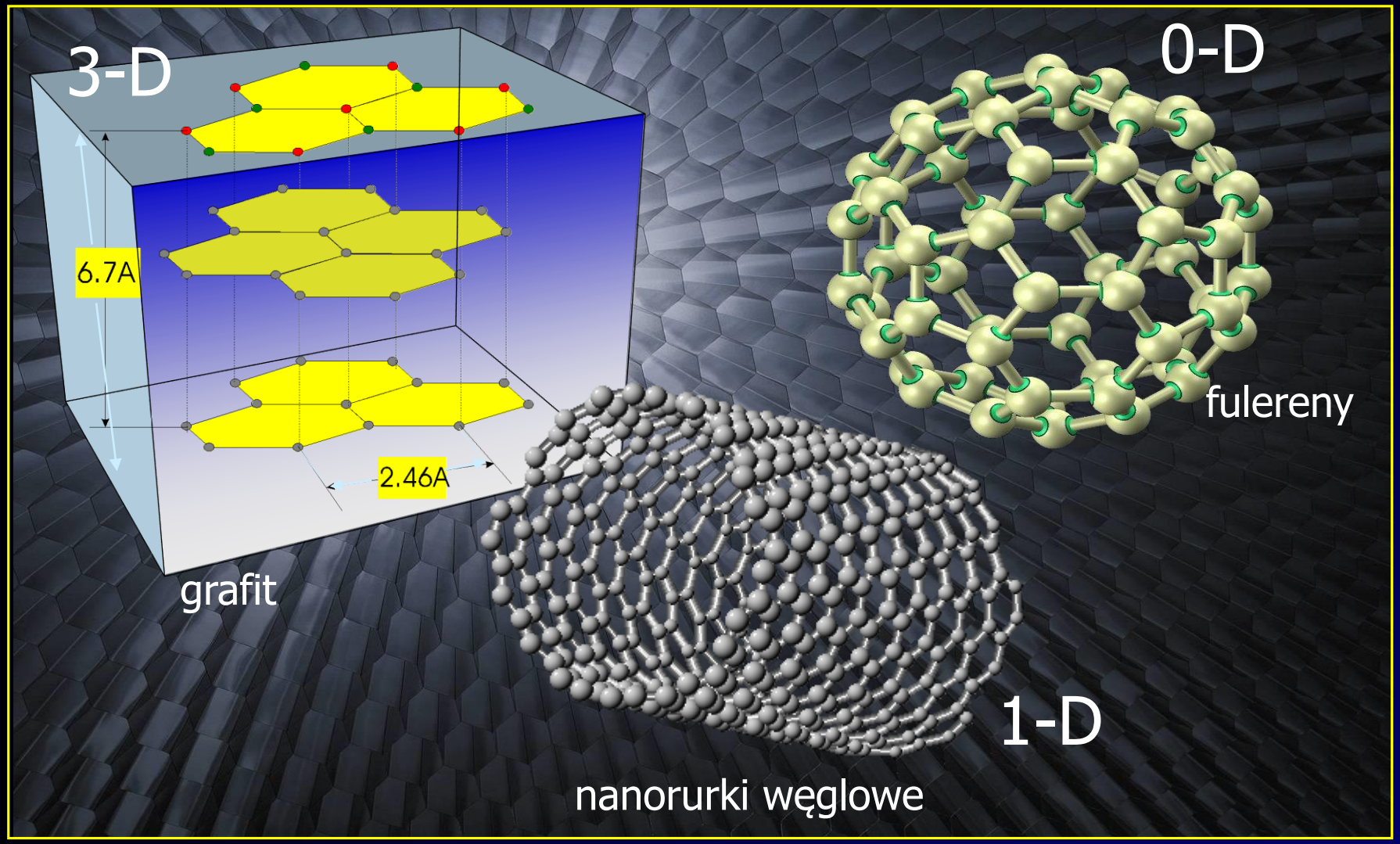
spin spintronika

pseudospin valleytronika

Nowe możliwości



Wymiarowość układów węglowych



Grafen 2-D

Kryterium Lindemanna

$$\left\langle (\delta R)^2 \right\rangle \sim R^2 \qquad R = \kappa a$$

70 lat temu Landau i Peierls pokazali teoretycznie
że struktury 2-D
są termodynamicznie niestabilne

L.D. Landau

R.E. Peierls

'Klątwa Landaua-Peierlsa'

R.E. Peierls, Helv. Phys. Acta 7 (1934) 61.
R.E. Peierls, Ann. I.H. Poincare 5 (1935) 177.
L.D. Landau, Phys. Z. Sowjetunion 11 (1937) 26.

Grafen 2-D

R.E. Peierls
L.D. Landau

W przybliżeniu harmonicznym, fluktuacje termiczne niszczą długozasięgowe uporządkowanie co prowadzi do procesu topnienia sieci 2-D w dowolnej temperaturze.

Kryterium Lindemanna

$$\left\langle (\delta R)^2 \right\rangle \sim R^2 \qquad R = \kappa a$$

R.E. Peierls, Helv. Phys. Acta 7 (1934) 61.
R.E. Peierls, Ann. I.H. Poincare 5 (1935) 177.
L.D. Landau, Phys. Z. Sowjetunion 11 (1937) 26.

N.D. Mermin, H. Wagner
N.D. Mermin

Długozasięgowe uporządkowanie magnetyczne nie istnieje w 1-D i 2-D.

Później wynik ten został rozszerzony na długozasięgowe uporządkowanie sieci krystalicznej (Mermin)

N.D. Mermin, H. Wagner, Phys. Rev. Lett. 17 (1966) 1133.
N.D. Mermin Phys. Rev. 176 (1968) 250.

Grafen 2-D

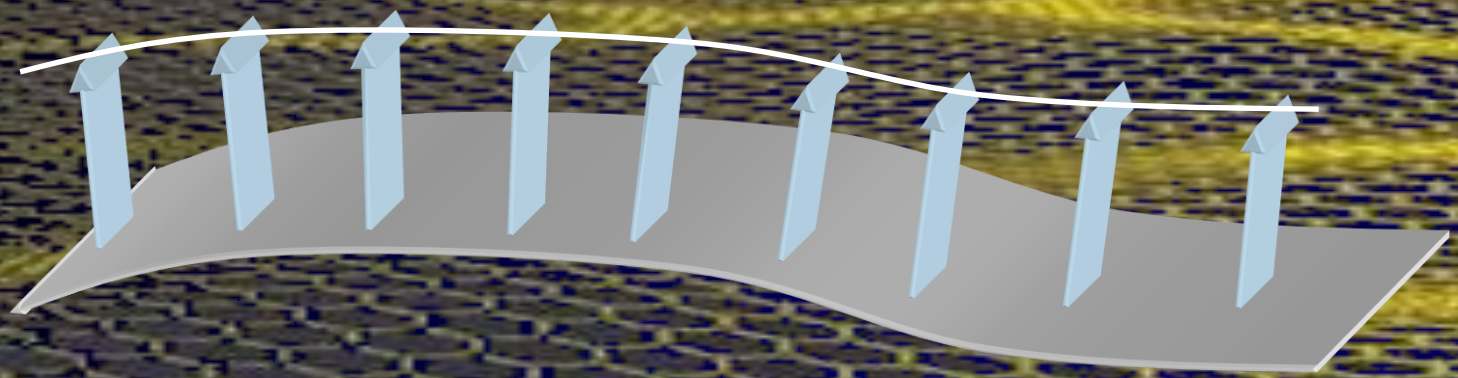
grafen nie istnieje

80 lat temu Landau i Peierls pokazali teoretycznie
że 2-D struktury są termodynamicznie niestabilne

Może jednak się mylili ?

2-D (grafen ?) w 3-D

Poza przybliżenie harmoniczne ...



Struktury 2-D w przestrzeni 3-D

D.R. Nelson, L. Peliti, J. Phys. 48 (1987) 1085.
L. Radzihovsky, P. Le Doussal, Phys. Rev. Lett. 69 (1992) 1209.
D.R. Nelson, T. Piran, S. Weinberg, Statiscical Mechanics of Membrans and Surfaces (World Scientific 2004)

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2-D (grafen ?) w 3-D

JARA Jülich Aachen Research Alliance

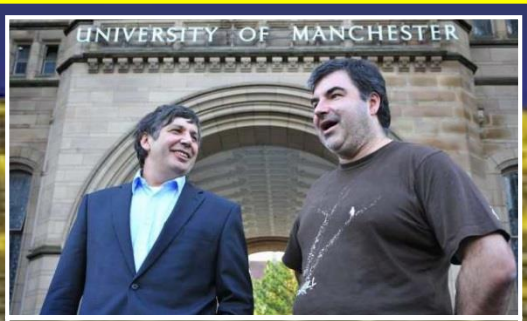
Grafen można wytworzyć !!!!!!!!

Vol 438|10 November 2005|doi:10.1038/nature04233 nature

LETTERS

Two-dimensional gas of massless Dirac fermions in graphene

K. S. Novoselov[1], A. K. Geim[1], S. V. Morozov[2], D. Jiang[1], M. I. Katsnelson[3], I. V. Grigorieva[1], S. V. Dubonos[2] & A. A. Firsov[2]



Andre Geim Kostya Novoselov

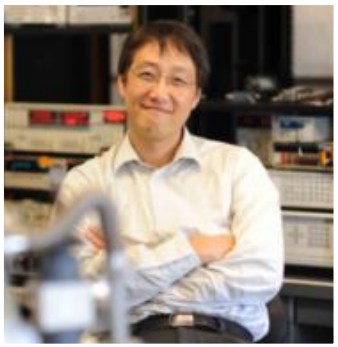
MANCHESTER 1824

Vol 438|10 November 2005|doi:10.1038/nature04235 nature

LETTERS

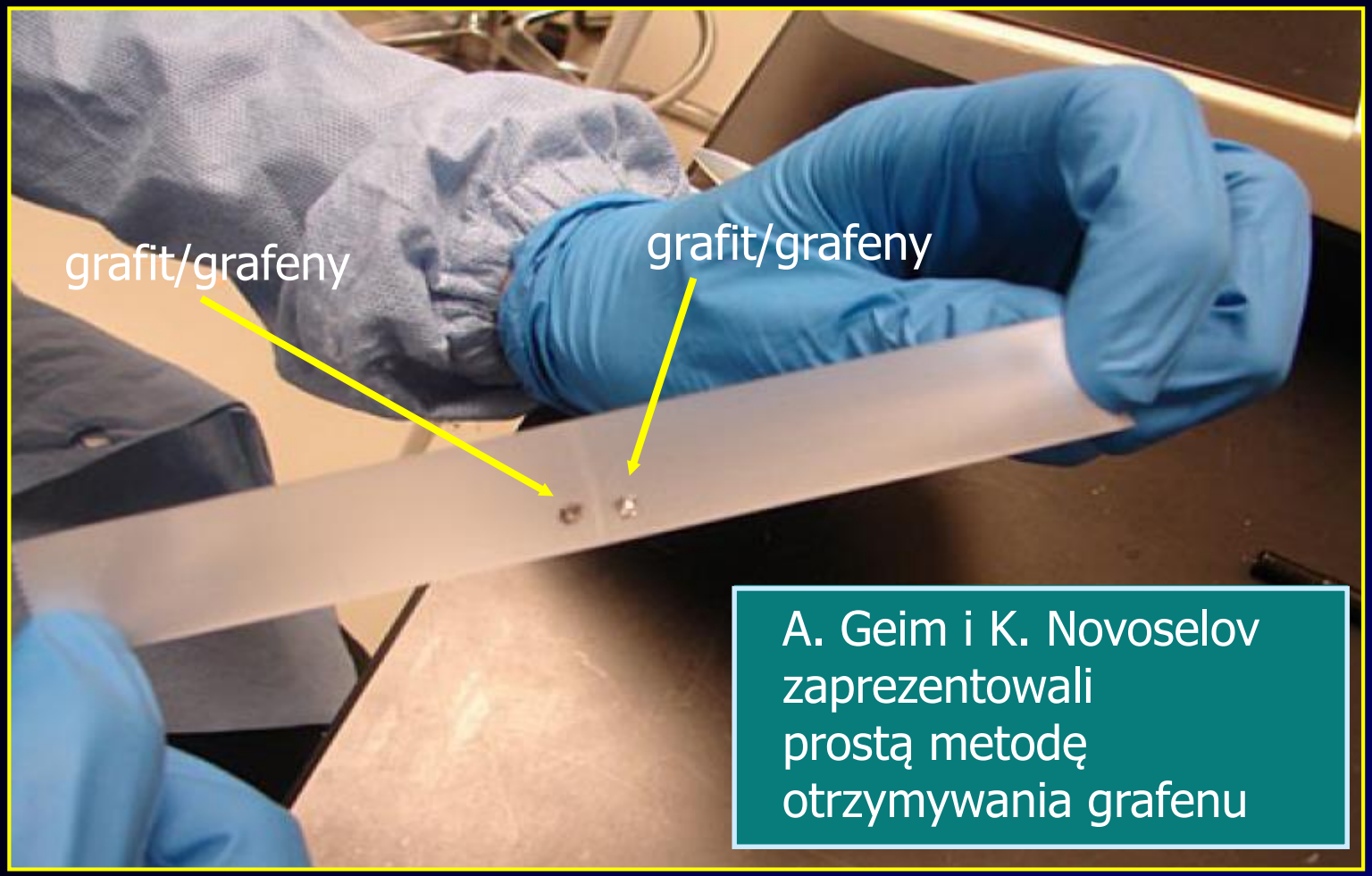
Experimental observation of the quantum Hall effect and Berry's phase in graphene

Yuanbo Zhang[1], Yan-Wen Tan[1], Horst L. Stormer[1,2] & Philip Kim[1]



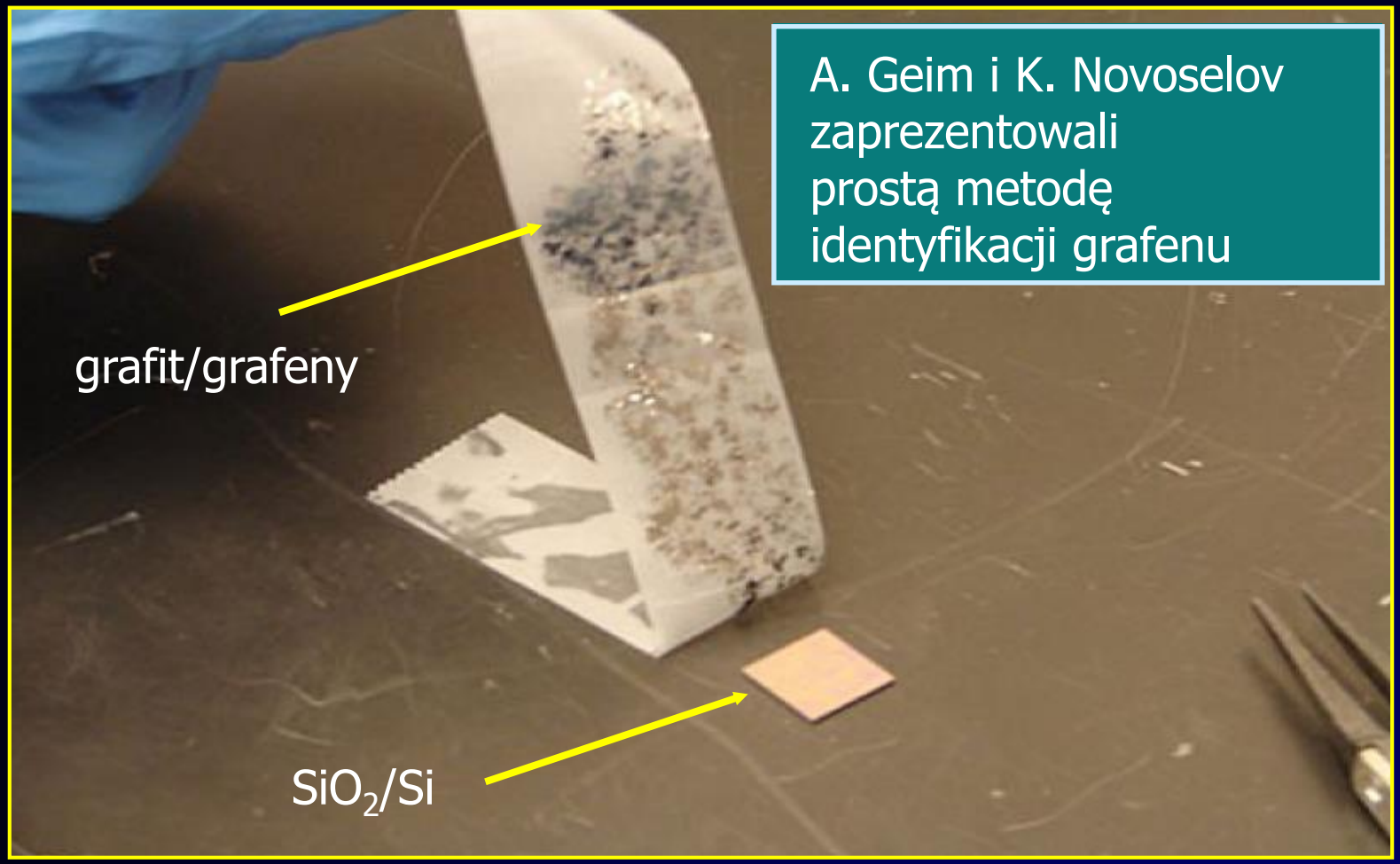
Kim Group
Department of Physics, Columbia University

Grafen można wytworzyć !!!!!!!!



A. Geim i K. Novoselov zaprezentowali prostą metodę otrzymywania grafenu

Grafen można wytworzyć !!!!!!!!



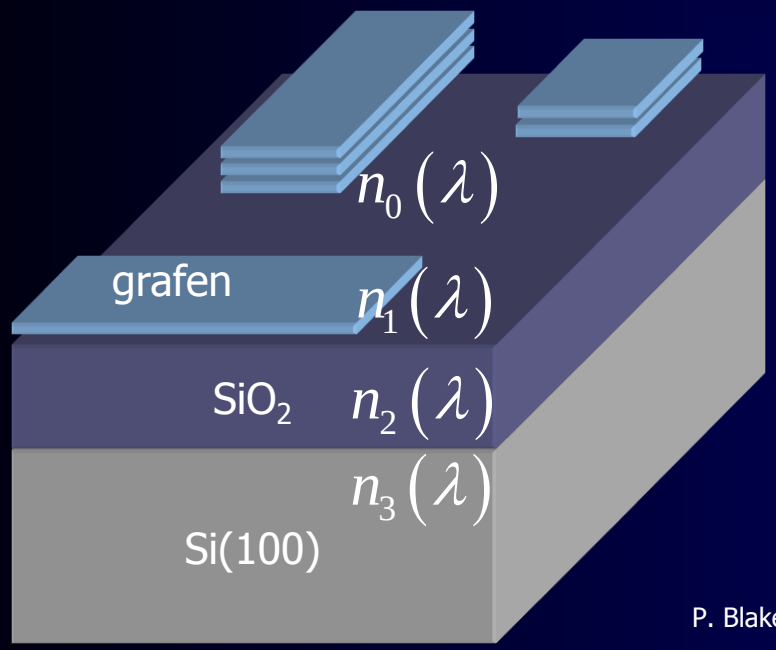
A. Geim i K. Novoselov zaprezentowali prostą metodę identyfikacji grafenu

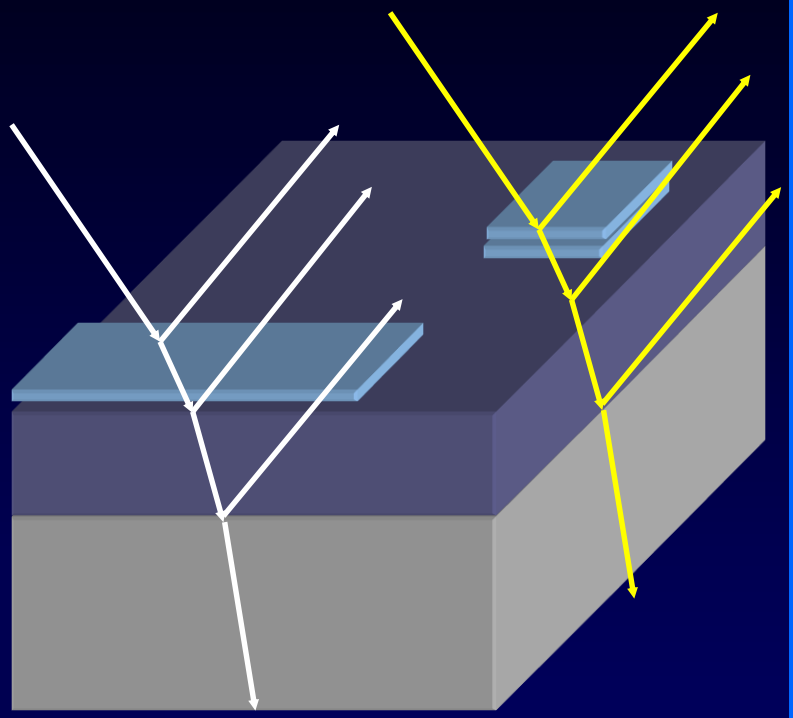
Grafen można zidentyfikować

Teoria Fresnela

grafen/SiO_2/Si

Współczynnik załamania

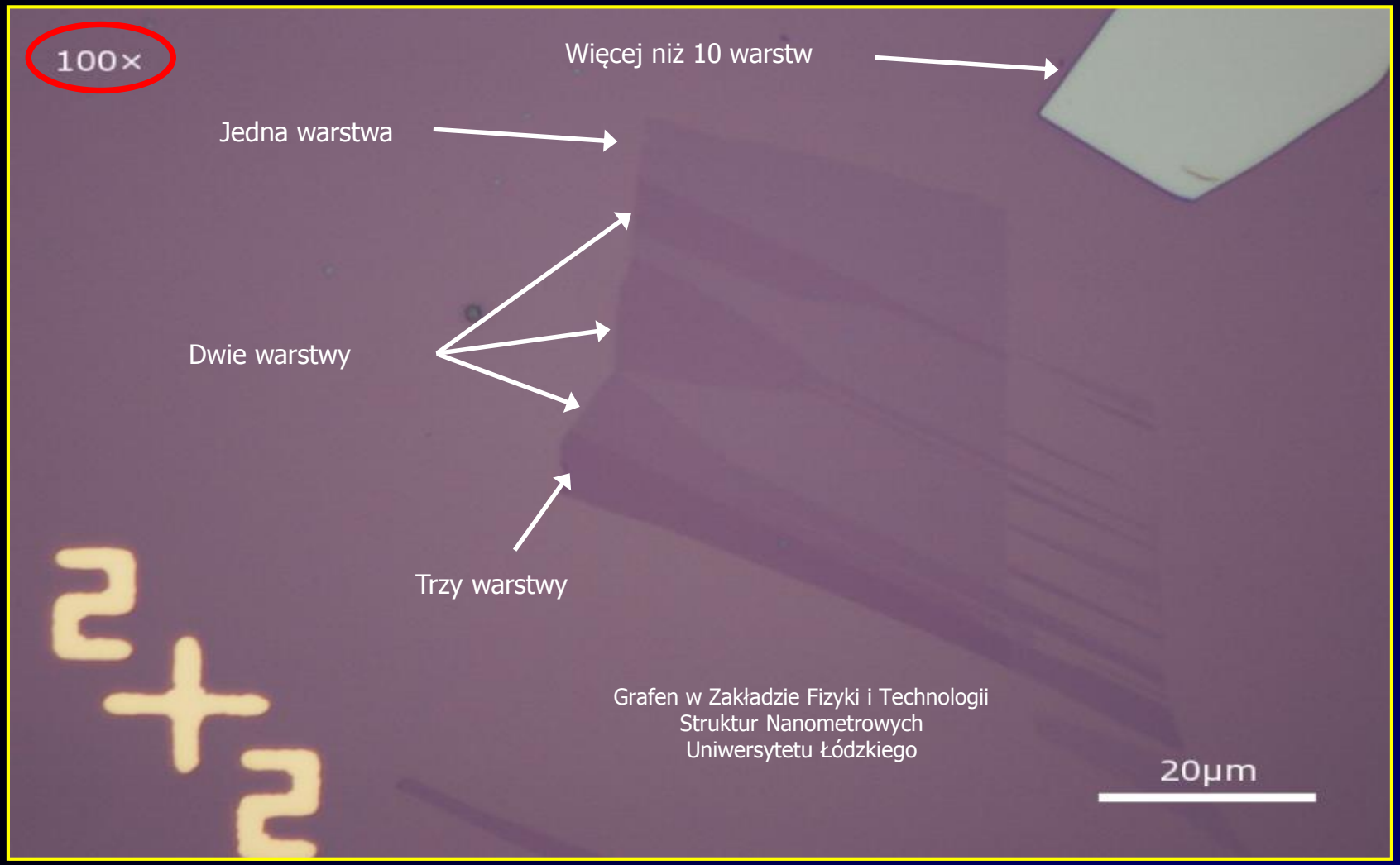




P. Blake, et al. Appl. Phys. Lett. 91, 063124 (2007).

I. Wlasny, P. Dabrowski, Z. Klusek, arXiv:1102.4953v1 (2011).

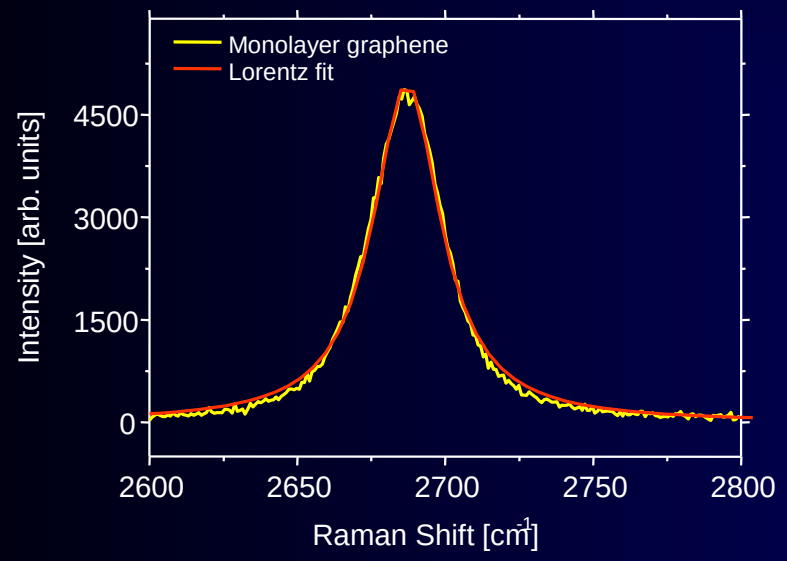
Grafen można zidentyfikować



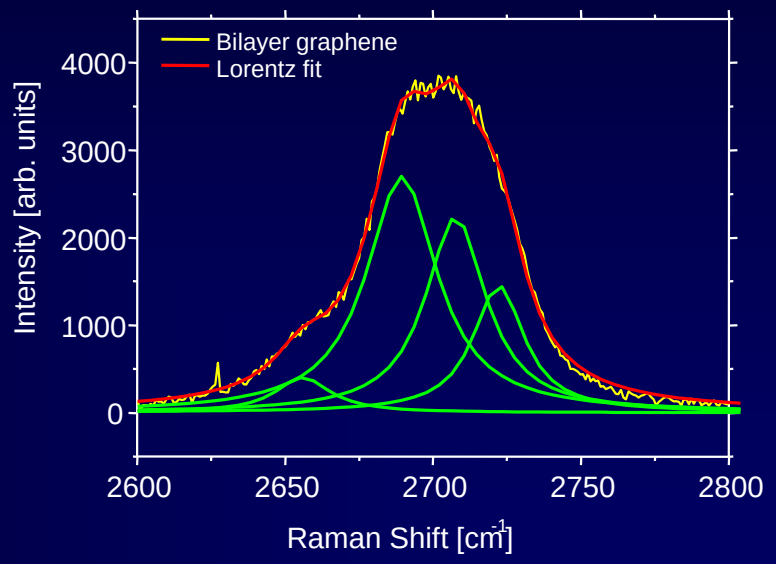
Grafen można zidentyfikować

2D pik – podwójny rezonans

Graphene/SiO_2/Si



Bilayer/SiO_2/Si



SiO_2

Grafen można zidentyfikować

Powszechna procedura identyfikacji

1 - warstwa - 1 komponent
2 - warstwy - 4 komponenty
3 - warstwy - 15 komponentów ->6

Więcej niż 5 warstw

2 komponenty jak w graficie

Grafen można zidentyfikować

UNIVERSITY OF CAMBRIDGE — DEPARTMENT OF ENGINEERING

Teoria/Eksperyment

PRL 97, 187401 (2006) PHYSICAL REVIEW LETTERS week ending 3 NOVEMBER 2006

Raman Spectrum of Graphene and Graphene Layers

A. C. Ferrari,[1,*] J. C. Meyer,[2] V. Scardaci,[1] C. Casiraghi,[1] M. Lazzeri,[3] F. Mauri,[3] S. Piscanec,[1] D. Jiang,[4] K. S. Novoselov,[4] S. Roth,[2] and A. K. Geim[4]

[1]*Cambridge University, Engineering Department, JJ Thompson Avenue, Cambridge CB3 0FA, United Kingdom*
[2]*Max Planck Institute for Solid State Research, Stuttgart 70569, Germany*
[3]*IMPMC, Universités Paris 6 et 7, CNRS, IPGP, 140 rue de Lourmel, 75015 Paris, France*
[4]*Department of Physics and Astronomy, University of Manchester, Manchester, M13 9PL, United Kingdom*

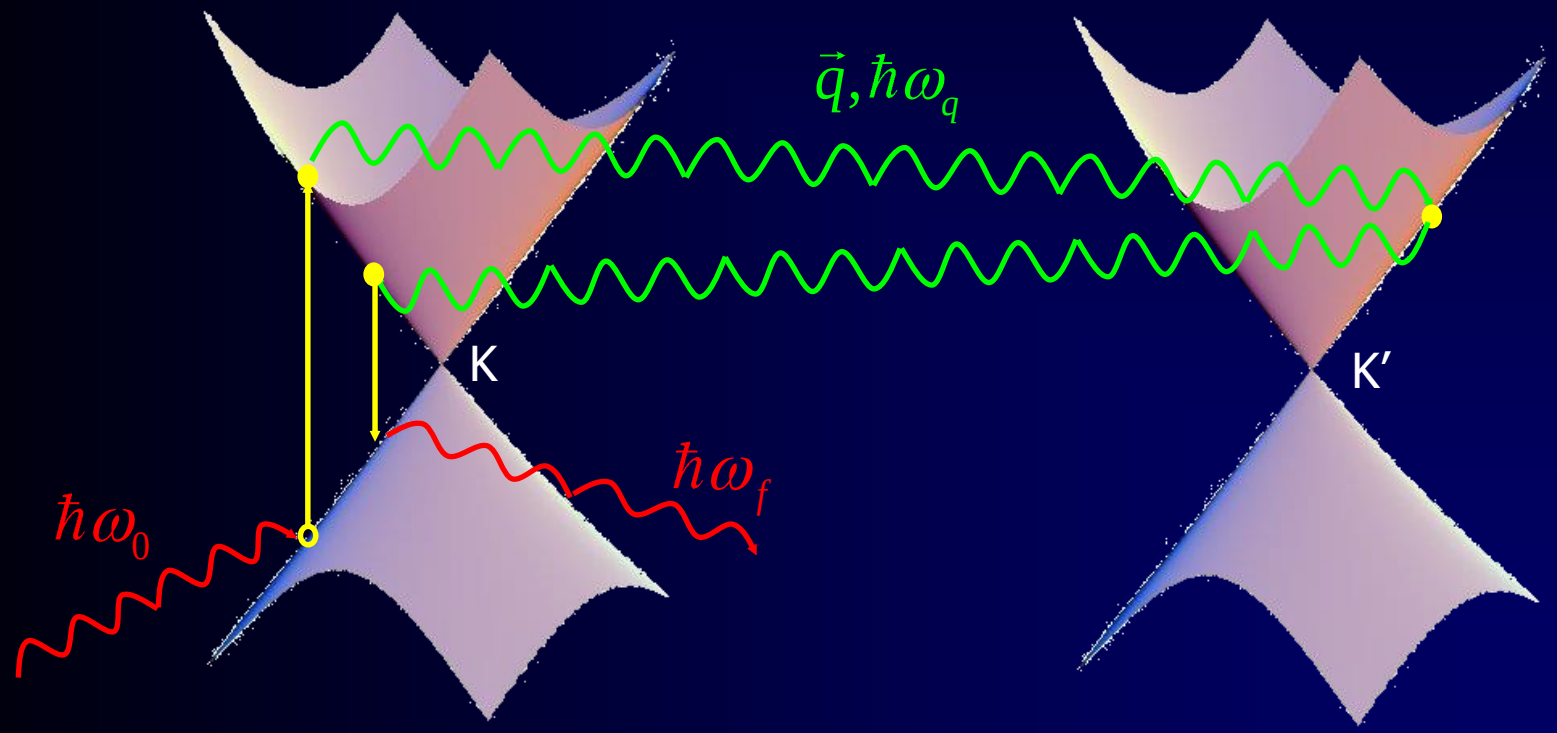
(Received 9 June 2006; published 30 October 2006)

[1,4]), in practice, it is only possible to distinguish between one and two layers by AFM if films contain folds or wrinkles [1,4]. This poses a major limitation to the range of substrates and is a setback for the widespread utilization of this material. Here, we show that graphene's electronic structure is uniquely captured in its Raman spectrum. Raman fingerprints for single layers, bilayers, and few layers reflect changes in the electron bands and allow unambiguous, high-throughput, nondestructive identification of graphene layers, which is critically lacking in this emerging research area.

A.C. Ferrari

Grafen można zidentyfikować

2D pik – podwójny rezonans



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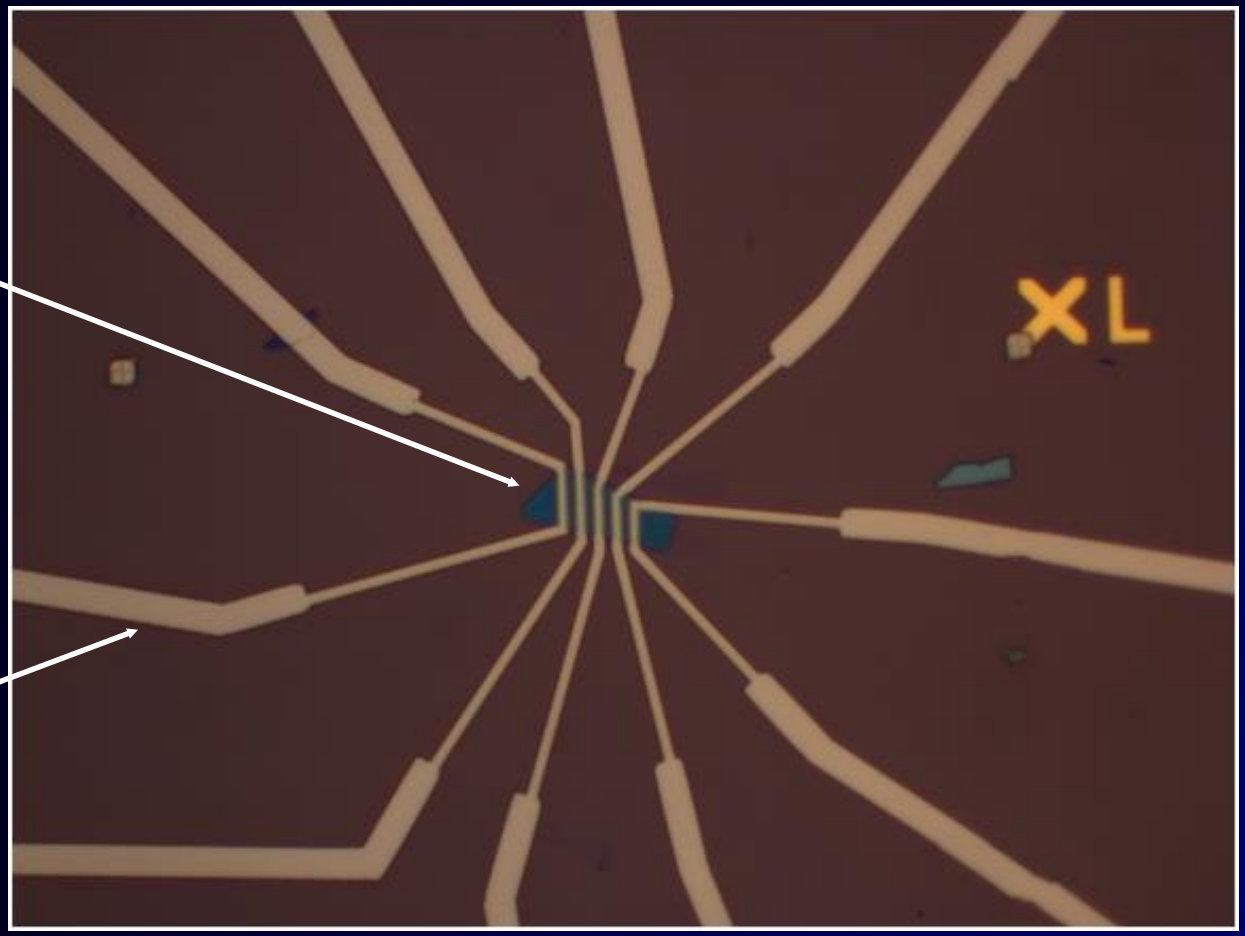
Grafen można zidentyfikować

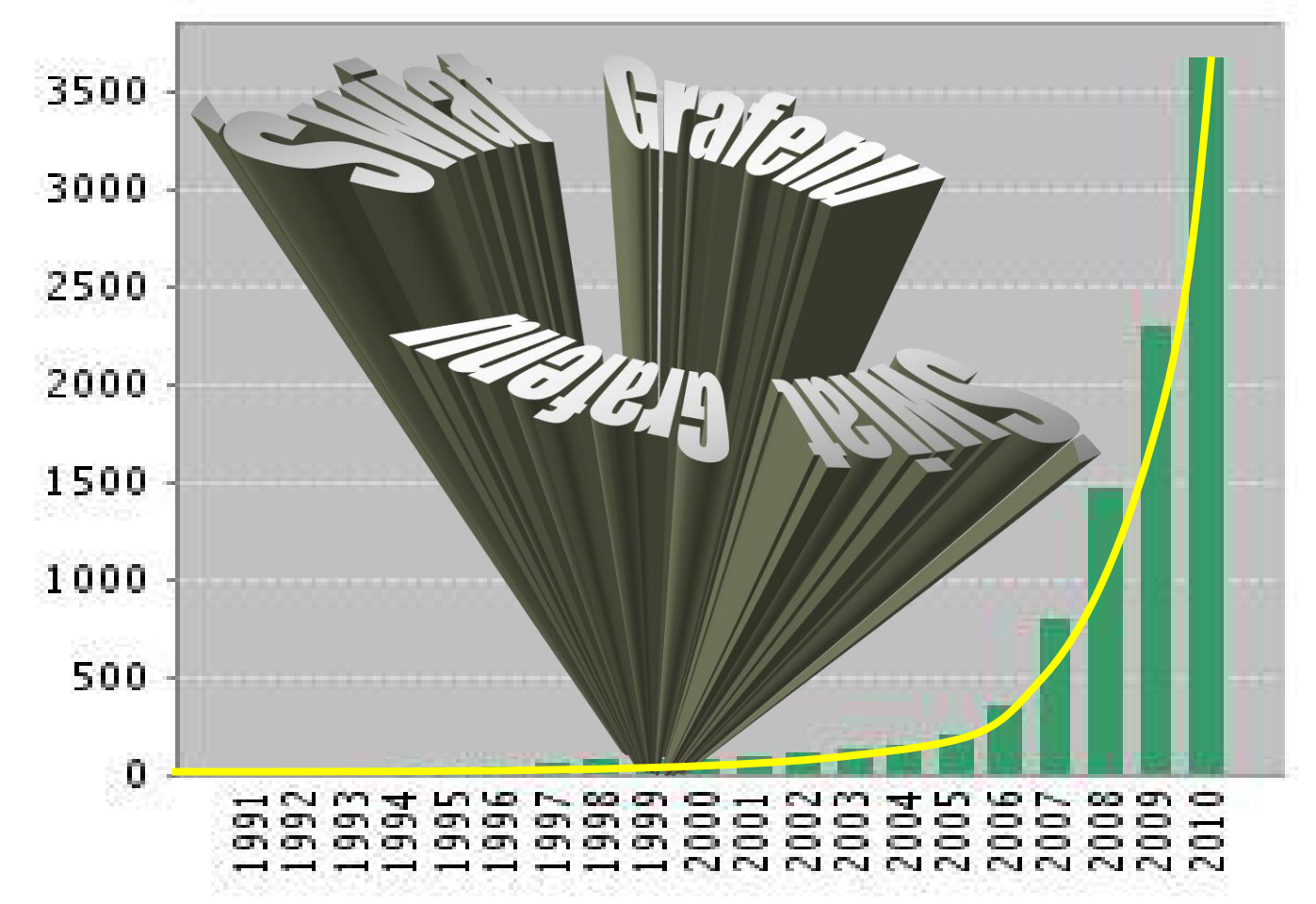
Zaczął się „bum” grafenowy

Wszystkich zafascynowały własności elektryczne grafenu



Zaczął się „bum” grafenowy

Zaczął się „bum” grafenowy



Zaczął się „bum” grafenowy

„for groundbreaking experiments regarding the two-dimensional material graphene"

Ig Nobel 2000
Nazwa pochodzi z języka angielskiego "ignoble" znaczy niegodziwy

The 2000 Ig Nobel Prize in Physics shared by Andrey Geim and Michael Berry
The Physics of Flying Frogs

Zaczął się „bum” grafenowy

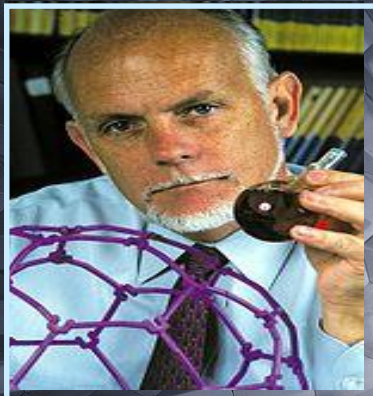
AntyNobel z Fizyki

Ig Nobel in Physics 2000

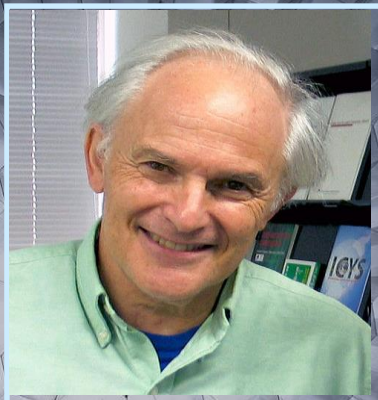
Zaczął się „bum” grafenowy

UNIWERSYTET ŁÓDZKI

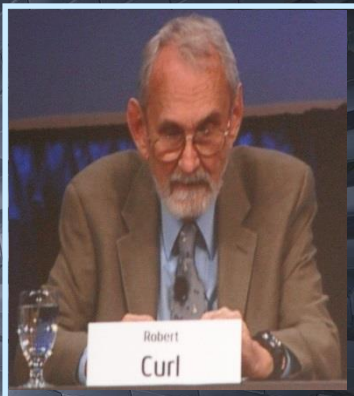
Zaczął się „bum” grafenowy



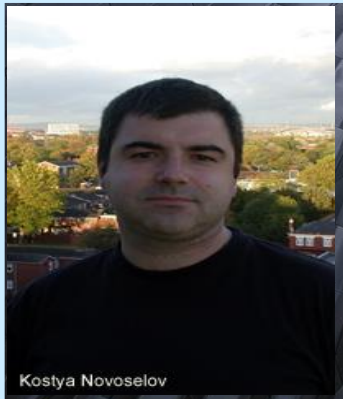
Richard Smalley / 1996



Harold Kroto / 1996

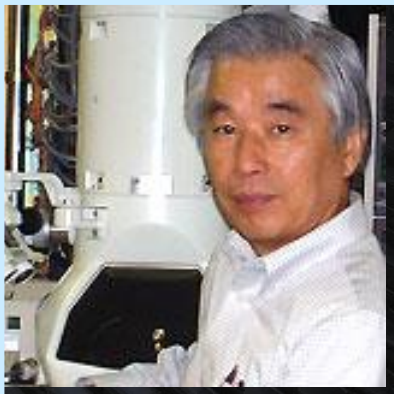


Robert Curl / 1996



Kostya Novoselov/2010

Andre Geim/2010



Sumio Iijima

Zaczął się „bum” grafenowy

Editorial by M. Monthioux, V. Kuznetsov [Carbon,44,1621,2006]

L. V. Radushkevich and V. M. Lukyanovich [Journal Phys. Chem., 1952]: 50 nm.

1952 ЖУРНАЛ ФИЗИЧЕСКОЙ ХИМИИ т. XXVI, вып. 1

О СТРУКТУРЕ УГЛЕРОДА, ОБРАЗУЮЩЕГОСЯ ПРИ ТЕРМИЧЕСКОМ РАЗЛОЖЕНИИ ОКИСИ УГЛЕРОДА НА ЖЕЛЕЗНОМ КОНТАКТЕ

Л. В. Радушкевич и В. М. Лукьянович

Historia

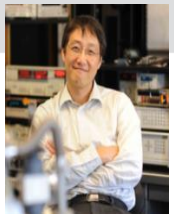
Vol 438|10 November 2005|doi:10.1038/nature04233 nature

LETTERS

Two-dimensional gas of massless Dirac fermions in graphene

K. S. Novoselov[1], A. K. Geim[1], S. V. Morozov[2], D. Jiang[1], M. I. Katsnelson[3], I. V. Grigorieva[1], S. V. Dubonos[2] & A. A. Firsov[2]

Vol 438|10 November 2005|doi:10.1038/nature04235 nature

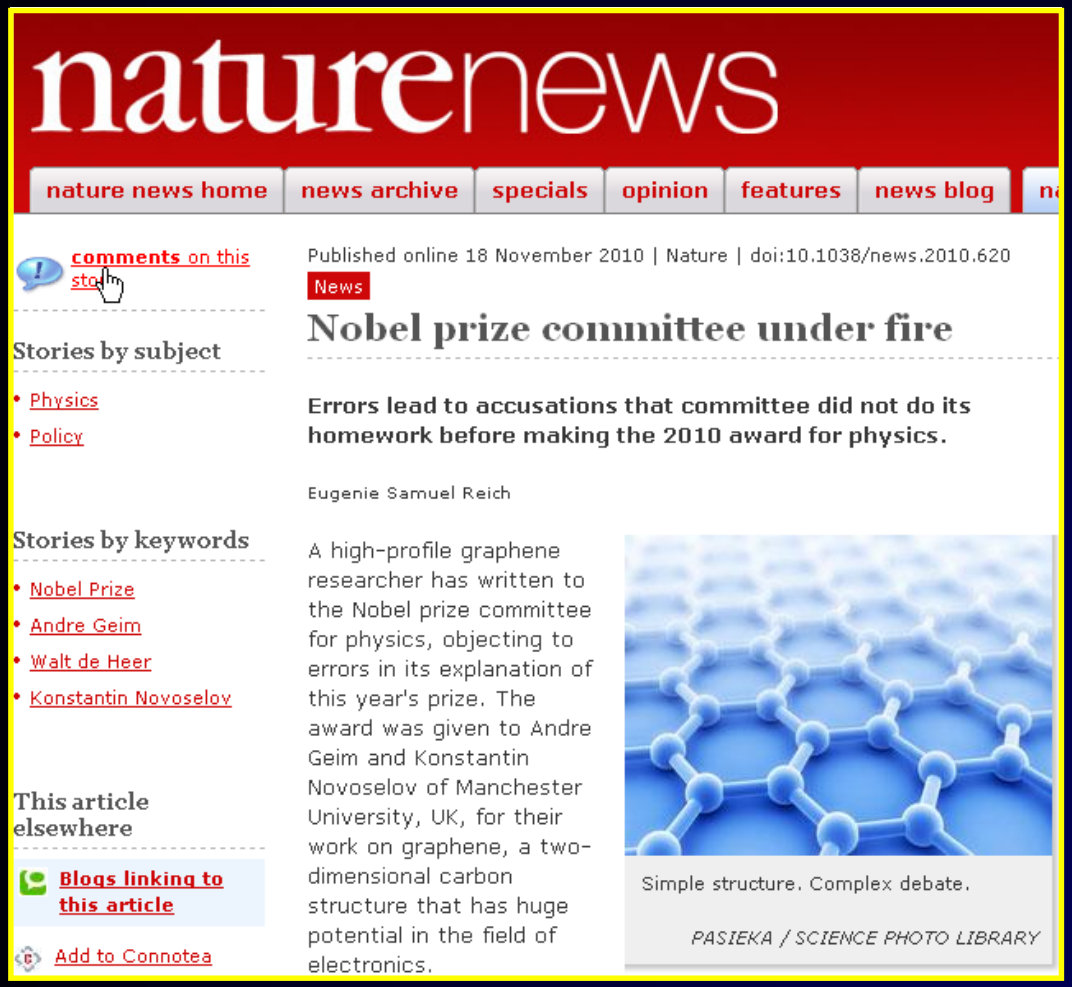


LETTERS

Experimental observation of the quantum Hall effect and Berry's phase in graphene

Yuanbo Zhang[1], Yan-Wen Tan[1], Horst L. Stormer[1,2] & Philip Kim[1]

Kontrowersje.....



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Stories by keywords

- Nobel Prize
- Andre Geim
- Walt de Heer
- Konstantin Novoselov

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Add to Connotea

Published online 18 November 2010 | Nature | doi:10.1038/news.2010.620

News

Nobel prize committee under fire

Errors lead to accusations that committee did not do its homework before making the 2010 award for physics.

Eugenie Samuel Reich

A high-profile graphene researcher has written to the Nobel prize committee for physics, objecting to errors in its explanation of this year's prize. The award was given to Andre Geim and Konstantin Novoselov of Manchester University, UK, for their work on graphene, a two-dimensional carbon structure that has huge potential in the field of electronics.

Simple structure. Complex debate.

PASIEKA / SCIENCE PHOTO LIBRARY

Walt de Heer
Georgia Institute of Technology, Atlanta

de Herr napisał list do Komitetu Noblowskiego

Kontrowersje.....

October 22
2004

Science
AAAS

K. S. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, Y. Zhang, S. V. Dubonos,
I. V. Grigorieva, and A. A. Firsov

Electric Field Effect in Atomically Thin Carbon Films

December 3
2004

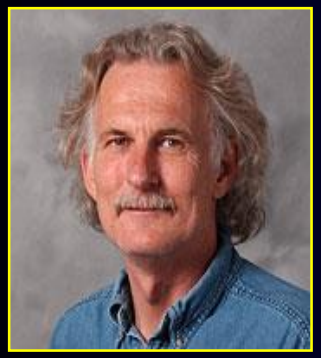
THE JOURNAL OF PHYSICAL CHEMISTRY B

J. Phys. Chem. B2004, vol. 108, 19912-19916

Claire Berger,[‡] Zhimin Song, Tianbo Li, Xuebin Li, Asmerom Y. Ogbazghi, Rui Feng, Zhenting Dai, Alexei
N. Marchenkov, Edward H. Conrad, Phillip N. First, and Walt A. de Heer

Ultrathin Epitaxial Graphite:
2D Electron Gas Properties and a Route toward Graphene-based Nanoelectronics

Kontrowersje.....



Walt de Heer
Georgia Institute of Technology, Atlanta

Discovery of graphene

Crystal structure	Debye P, Scherrer P (1916).
Graphite band structure calculation	Wallace (1947) (includes "Dirac" cone)
Isolation and observation of freestanding graphene	Boehm et al (1962)
Official naming of graphene	Boehm et al (1987)

Graphene on transition metals, carbides

Identification of:
"monolayer of graphite"
"single-crystal plane"
"two-dimensional graphite"

SiC	Van Bommel, Surf. Sci. (1975)
LaB6	Oshima Appl Phys (1977)
Pt	Zi-Du Surface Science (1987)...
Ni	Rosei PRB(1983)
Ir	Kholin Surf Sci (1984)
Re	Gall Sov Phys Sol State (1985)
TaC	Aizawa PRL 1990
TiC	Nagashima, Surf Sci (1993)
Ru	Marchini (2007)
WC	TaC, HfC,...
SiC	Forbeaux (1998)

Graphene-based electronics conceived

Gateable graphenes
Emphasis on transport

Georgia Tech (2001)

SiO_2	Novoselov Nature (2004)
	Transport on transferred exfoliated few layer graphite observation of graphene (GOOD FOR 2D PHYSICS)
SiC	Berger J. Chem Phys (2004)
Epitaxial	Transport on graphene and few layer graphite (GOOD FOR ELECTRONICS)

Kontrowersje.....

Eksfoljacja mechaniczna

Manchester, Science ’04

Grube warstwy ?
Kurtz PRB1990
Ebbesen Adv Mat 1995
Ohashi Tanso 1997
Ruoff APL1999
Gan Surf Sci2003

Metoda CVD

McConville 1986 (on Ni)
Land 1992 (on Pt)

Metoda transferu:
Geim & Novoselov
Nature Mat. (2007)

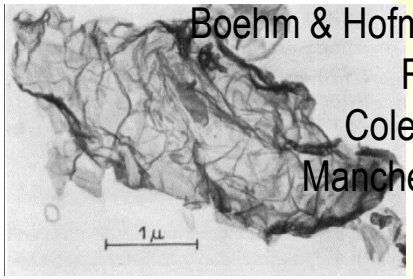
Realizacja :
MIT (2008)
Yu(2008)
Hong (2009)
Ruoff (2009)

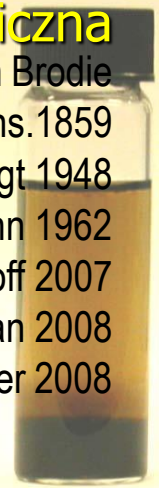
Epitaksjalny wzrost na SiC

Bommel 1975
Nagashima 1993
Forbeaux 1998
de Heer 2004
HRL 2009
IBM 2009

Eeksfoljacja chemiczna

Benjamin Brodie
Phil Trans.1859
Ruess & Vogt 1948
Boehm & Hofmann 1962
Ruoff 2007
Coleman 2008
Manchester 2008





Kontrowersje.....

NSF Proposal , 2001
Patterned Graphite Nanoelectronics

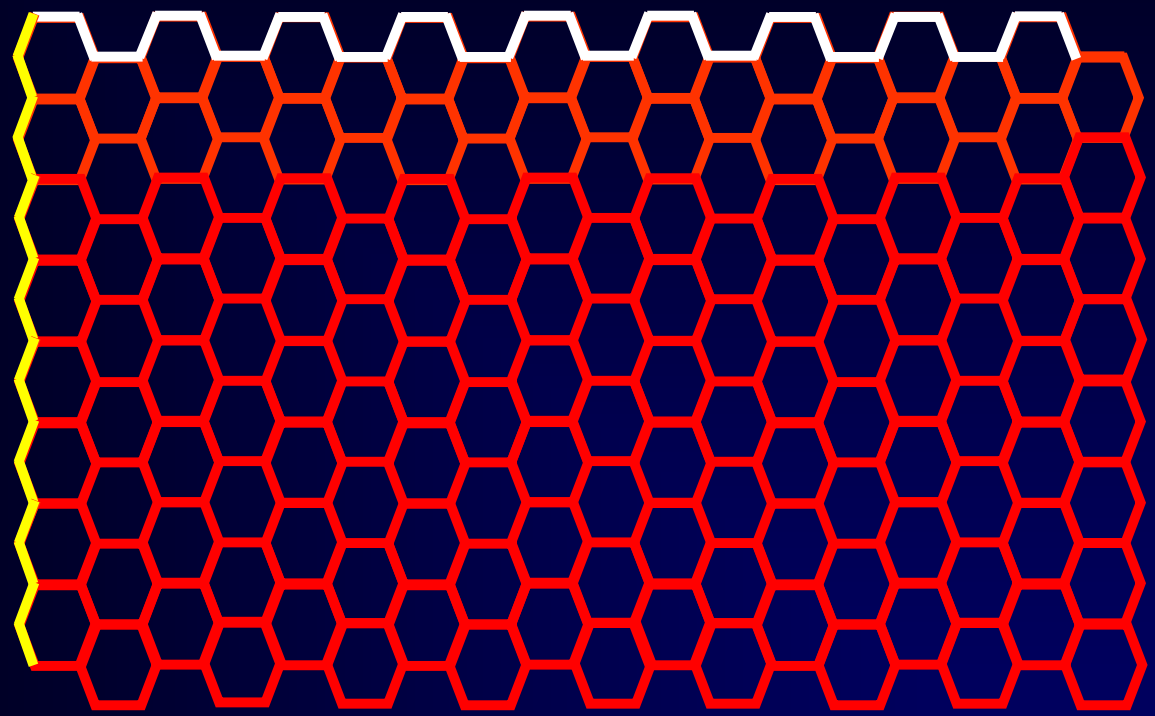
3 cytowania naszej pracy w NSF Proposal, 2001
w różnych aspektach grantu

Z. Klusek, Z. Waqar, E. Denisov, T. Kompaniets, I. Makarenko,
A. Titkov, and A. Bhatti
Appl. Surf. Sci., vol. 161, p. 508, 2000.

Kontrowersje.....

teoria

armchair

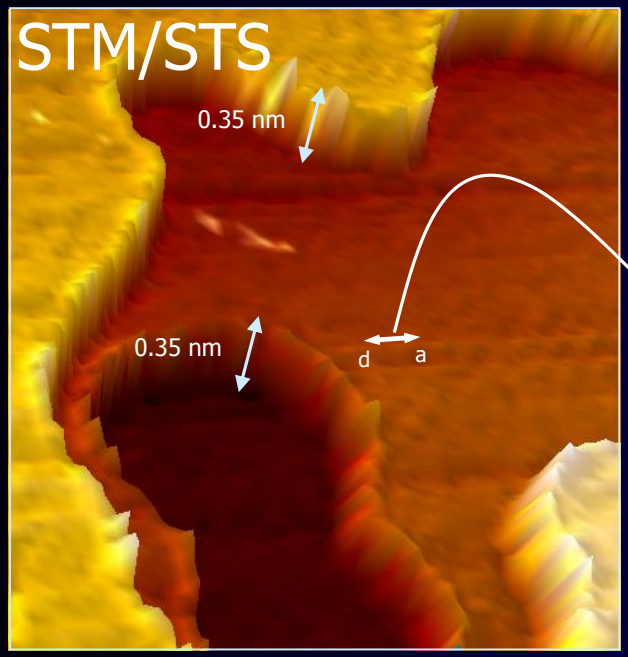


zig-zag

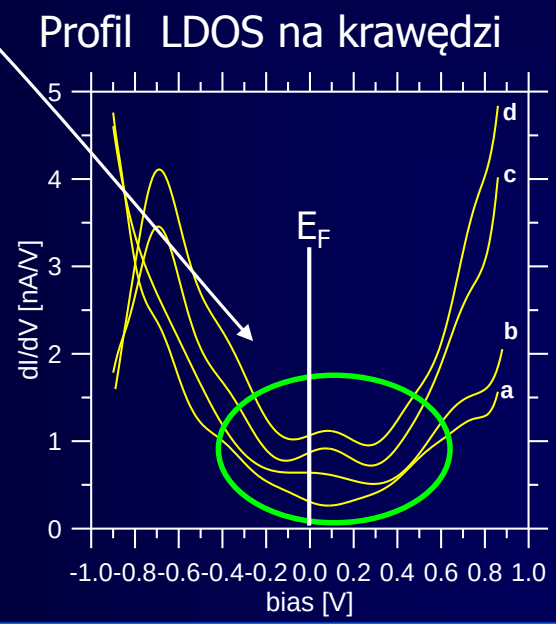
K. Kobayashi	Phys. Rev. B 48 (1993) 1757.
R. Tamura, M. Tsukada	Phys. Rev. B 49 (1994) 7697.
M. Fujita, K. Wakabayashi, K. Nakada, K. Kusakabe	J. Phys. Soc. Jpn. 65 (1996) 1920.
K. Nakada, M. Fujita, G. Dresselhaus, M. Dresselhaus	Phys. Rev. B 54 (1996) 17954.
K. Wakabayashi, M. Fujita, H. Ajiki, M. Sigrist	Phys. Rev. B 59 (1999) 8271.
R. Tamura, K. Akagi, M. Tsukada, S. Itoh, S. Ihara	Phys. Rev. B 56 (1999) 1404.
Y. Miyamoto, K. Nakada, M. Fujita	Phys. Rev. B 59 (1999) 9858.

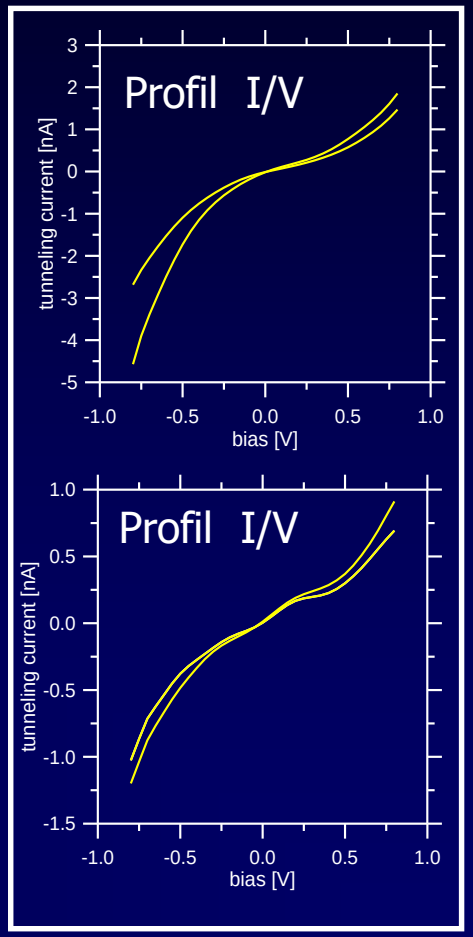
Kontrowersje.....

Z. Klusek, Z. Waqar, E. Denisov, T. Kompaniets, I. Makarenko, A. Titkov, A. Bhatti, Appl. Surf. Sci. 161, pp. 508-514, (2000).
Z. Klusek, Z. Waqar, W. Kozlowski, P. Kowlaczyk, E. Denisov, et al. Appl. Surf. Sci. 187, pp. 28-36, (2002).
Z. Klusek, P.K. Datta, W. Kozlowski, Corr. Sci. 45, pp. 1383-1393, (2003).
Z. Klusek, W. Kozlowski, P. Kowalczyk, A. Busiakiewicz, W. Olejniczak, et al. Acta Phys. Super. 6, pp. 33-42, (2004).



Początek badań na Uniwersytecie Łódzkim 1999 rok





Kontrowersje.....

Z. Klusek, Appl. Surf. Sci. 108, pp.405-414, (1997).
Z. Klusek, Appl. Surf. Sci. 125, pp.339-350, (1998).
Z. Klusek, Appl. Surf. Sci. 151, pp.251-261, (1999).
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Z. Klusek, Vacuum 63, (2001) 139.
Z. Klusek, et al. Appl. Surf. Sci. 187, pp. 28-36, (2002).
Z. Klusek, et al. Corr. Sci. 45, pp. 1383-1393, (2003).
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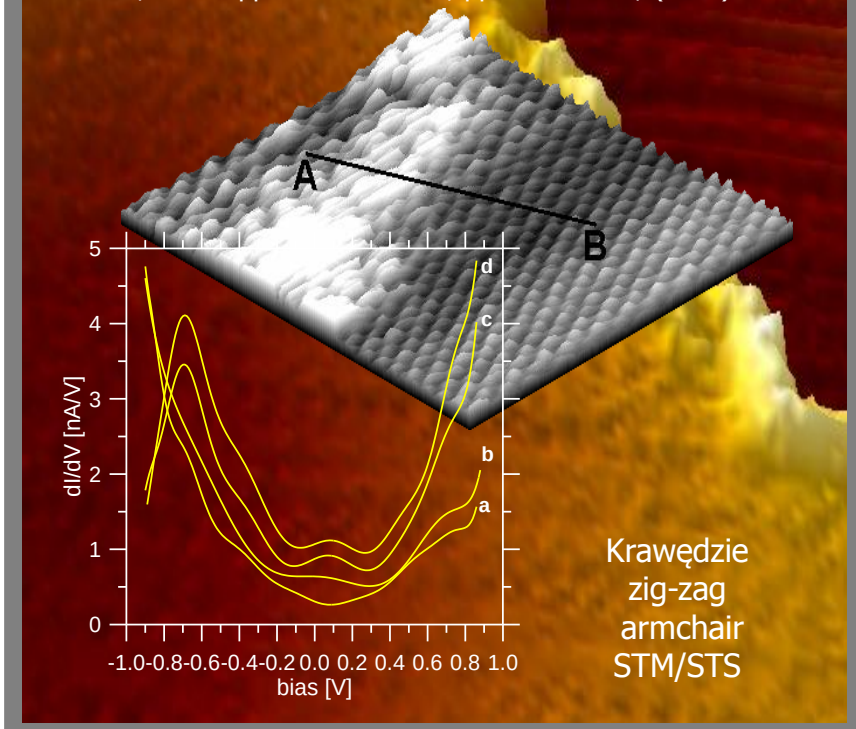
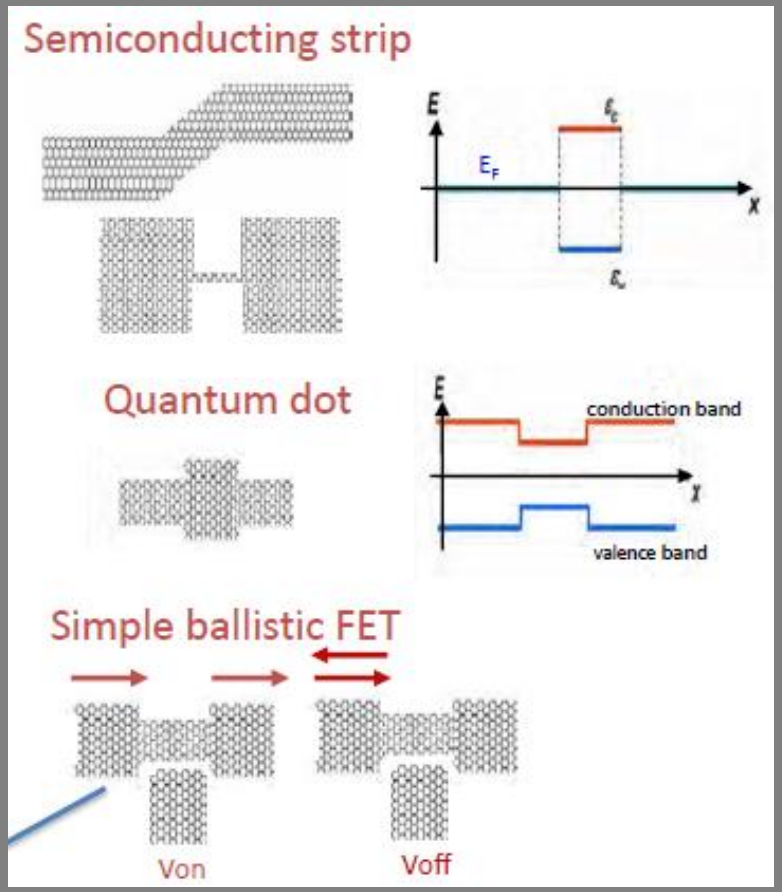
Z. Klusek, et al. Appl. Surf. Sci. 252, pp. 1221-1227, (2005).



Krawędzie
zig-zag
armchair
STM/STS

NSF Proposal , 2001
Patterned Graphite Nanoelectronics



Kontrowersje.....

NSF Proposal , 2001
Patterned Graphite Nanoelectronics

Referee responses: “not interesting”; “not possible”; “premature”

- W latach 2002-2003 udowodnili, że to możliwe
- Grant z Intela w 2003 oraz grant z NSF w 2004
- Patent w 2003

Kontrowersje.....

Confinement Controlled Sublimation

United States Patent 7,015,142

DeHeer, et al. March 21, 2006

Patterned thin film graphite devices and method for making same

Abstract

In a method of making graphite devices, a preselected crystal face of a crystal is annealed to create a thin-film graphitic layer disposed against selected face. A preselected pattern is generated on the thin-film graphitic layer. A functional structure includes a crystalline substrate having a preselected crystal face. A thin-film graphitic layer is disposed on the preselected crystal face. The thin-film graphitic layer is patterned so as to define at least one functional structure.

Inventors: ***DeHeer*; Walt A.** (Atlanta, GA), ***Berger*; Claire** (Atlanta, GA), **First; Phillip N.** (Doraville, GA)

Assignee: **Georgia Tech Research Corporation** (Atlanta, GA)

Appl. No.: **10/860,710**

Filed: **June 3, 2004**

Related U.S. Patent Documents

Application Number	Filing Date	Patent Number	Issue Date
60477997	Jun., 2003		

First CCS Induction Furnace

(2004)

Kontrowersje.....

Science 22 October 2004: Vol. 306 no. 5696 pp. 666-669

Science AAAS

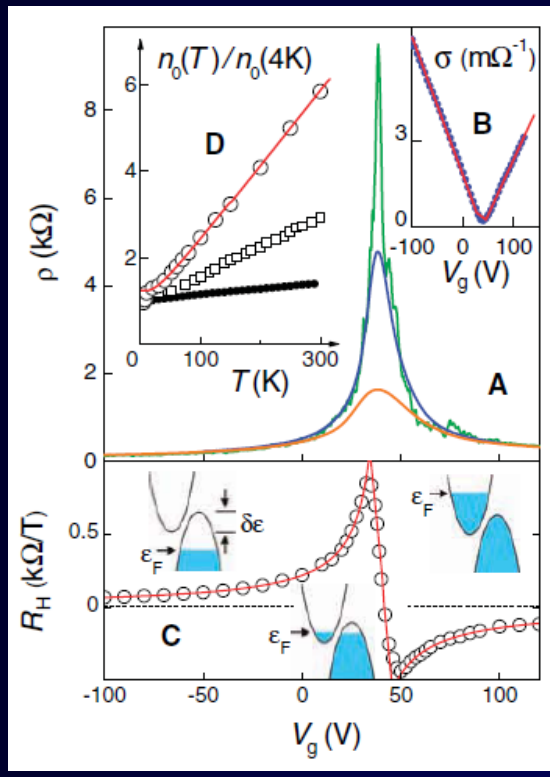


Fig. 2. Field effect in FLG. (A) Typical dependences of FLG's resistivity ρ on gate voltage for different temperatures (T = 5, 70, and 300 K for top to bottom curves, respectively). (B) Example of changes in the film's conductivity $\sigma = 1/\rho(V_g)$ obtained by inverting the 70 K curve (dots). (C) Hall coefficient R_H versus V_g for the same film; T = 5 K. (D) Temperature dependence of carrier concentration n_0 in the mixed state for the film in (A) (open circles), a thicker FLG film (squares), and multi-layer graphene ($d \approx 5$ nm; solid circles). Red c[illegible] in (B) to (D) are the depend[illegible] calculated from our model of a[illegible] semimetal illustrated by insets i[illegible].

Kontrowersje.....

Vol 438|10 November 2005|doi:10.1038/nature04233 nature

LETTERS

Two-dimensional gas of massless Dirac fermions in graphene

K. S. Novoselov[1], A. K. Geim[1], S. V. Morozov[2], D. Jiang[1], M. I. Katsnelson[3], I. V. Grigorieva[1], S. V. Dubonos[2] & A. A. Firsov[2]

Vol 438|10 November 2005|doi:10.1038/nature04235 nature

LETTERS

Experimental observation of the quantum Hall effect and Berry's phase in graphene

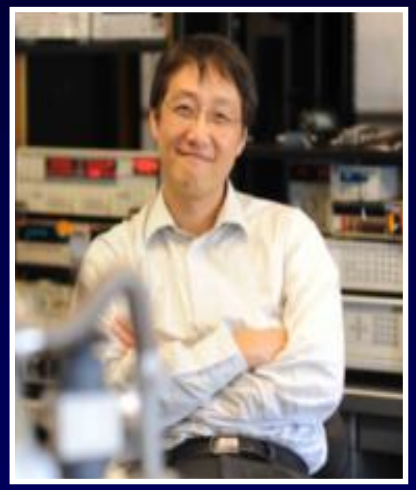
Yuanbo Zhang[1], Yan-Wen Tan[1], Horst L. Stormer[1,2] & Philip Kim[1]

Kontrowersje.....

A może Phil Kim powinien być również laureatem Nagrody Nobla z Fizyki za 2010 ?

Kostia Novoselov

Andre Geim



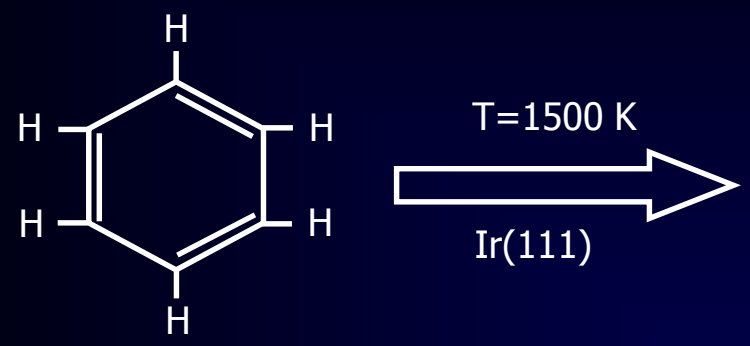
Phil Kim

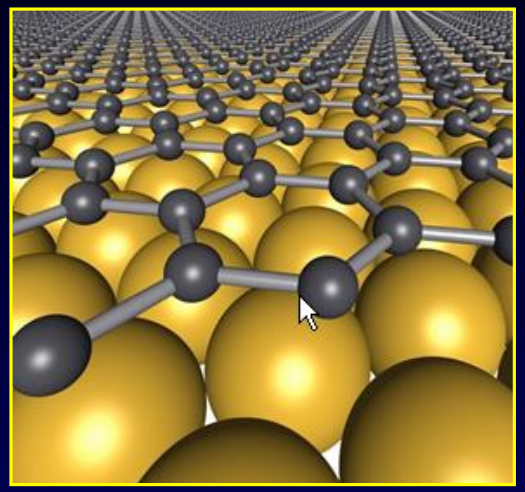
The high-mobility graphene samples used in our experiments were extracted from Kish graphite (Toshiba Ceramics) on degenerately doped Si wafers with a 300-nm SiO_2 coating layer, by using micromechanical manipulation similar to that described in ref. 8.

8. Novoselov, K. S. *et al.* Electric field effect in atomically thin carbon films. *Science* 306, 666–669 (2004).

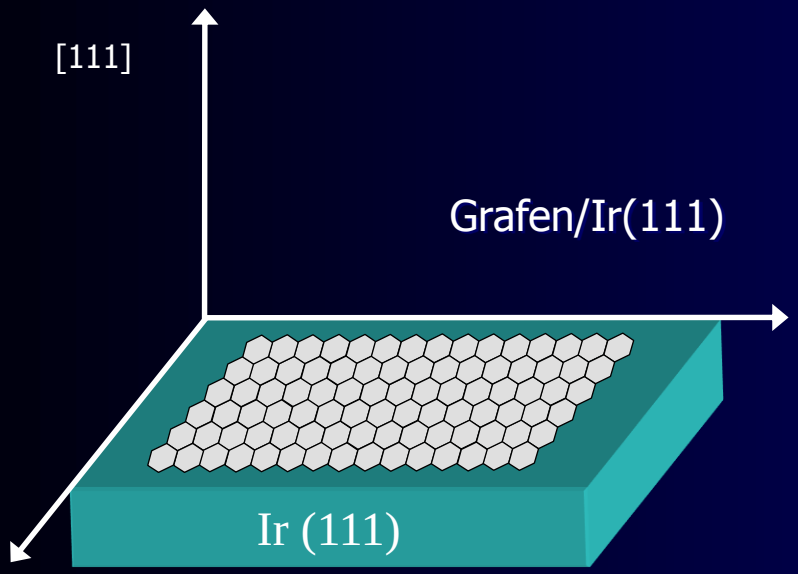
Grafen na Uniwersytecie Łódzkim
20 lat minęło

Grafen na Uniwersytecie Łódzkim





Początek badań w Łodzi
1999 rok

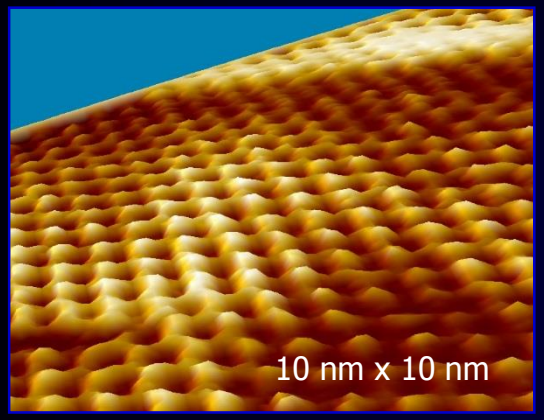


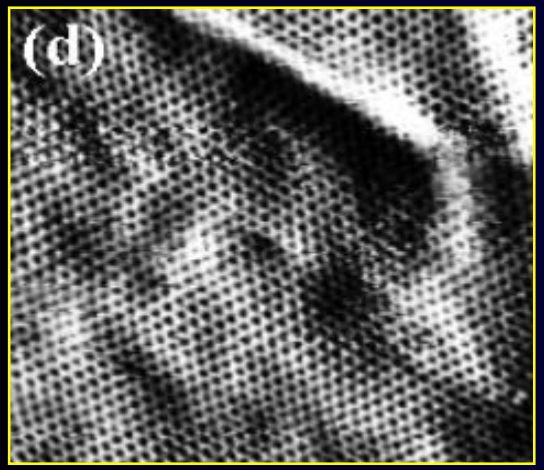
A. F. Ioffe Physical-Technical Institute
Russian Academy of Sciences
St. Petersburg, Russia

Solid State Electronics Department, Research Institute of Physics
St. Petersburg University
St. Petersburg, Russia

UNIWERSYTET ŁÓDZKI

Grafen na Uniwersytecie Łódzkim





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Local electronic edge states of graphene layer deposited on Ir(1 1 1) surface studied by STM/CITS

Z. Klusek [a,b,*], W. Kozlowski [a,b], Z. Waqar [c,d], S. Datta [b], J.S. Burnell-Gray [b], I.V. Makarenko [c], N.R. Gall [c], E.V. Rutkov [c], A.Ya. Tontegode [c], A.N. Titkov [c]

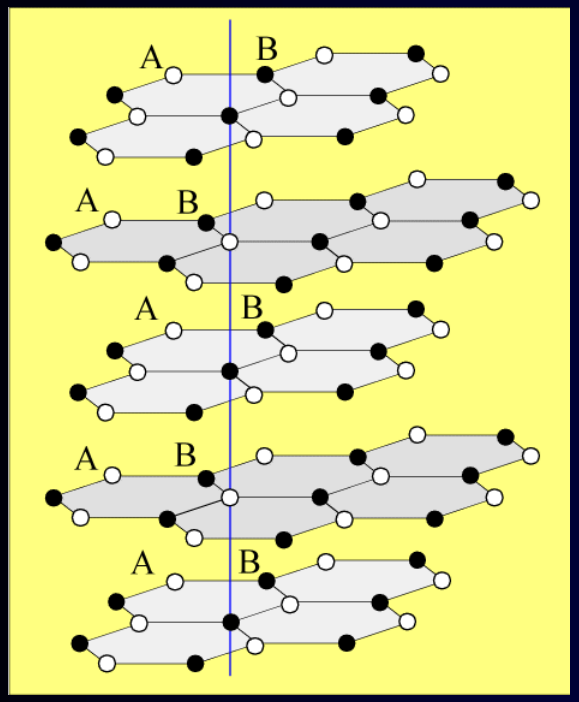
[a] *Department of Solid State Physics, University of Lodz, 90-236 Lodz, Pomorska 149/153, Poland*
[b] *Advanced Materials Research Institute, University of Northumbria, Ellison Building, Ellison Place, Newcastle upon Tyne NE18ST, UK*
[c] *A.F. Ioffe Physical-Technical Institute, Russian Academy of Sciences 26, Politekhnicheskaya 194021, St. Petersburg, Russia*
[d] *Lash Miller Chemical Laboratories, Department of Chemistry, University of Toronto, Ont., Canada M5S 3H6*

Received 15 September 2004; received in revised form 7 February 2005; accepted 8 February 2005
Available online 19 March 2005

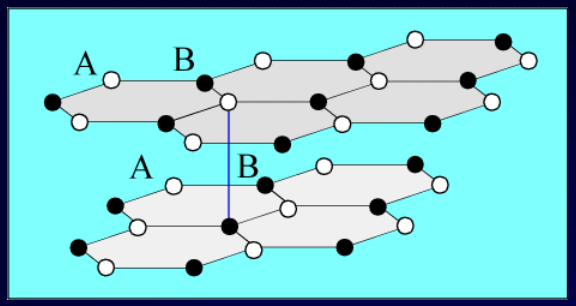
Pierwsza próba opublikowania rezultatów w roku 2000 nie powiodła się

Grafen

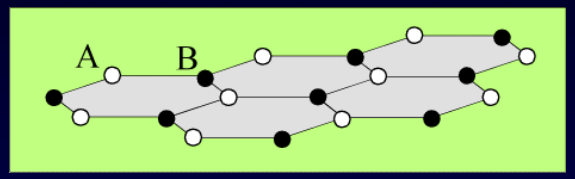
grafit



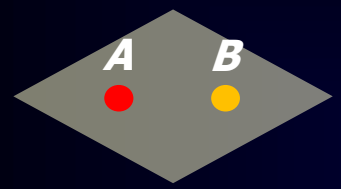
dwuwarstwa grafenowa



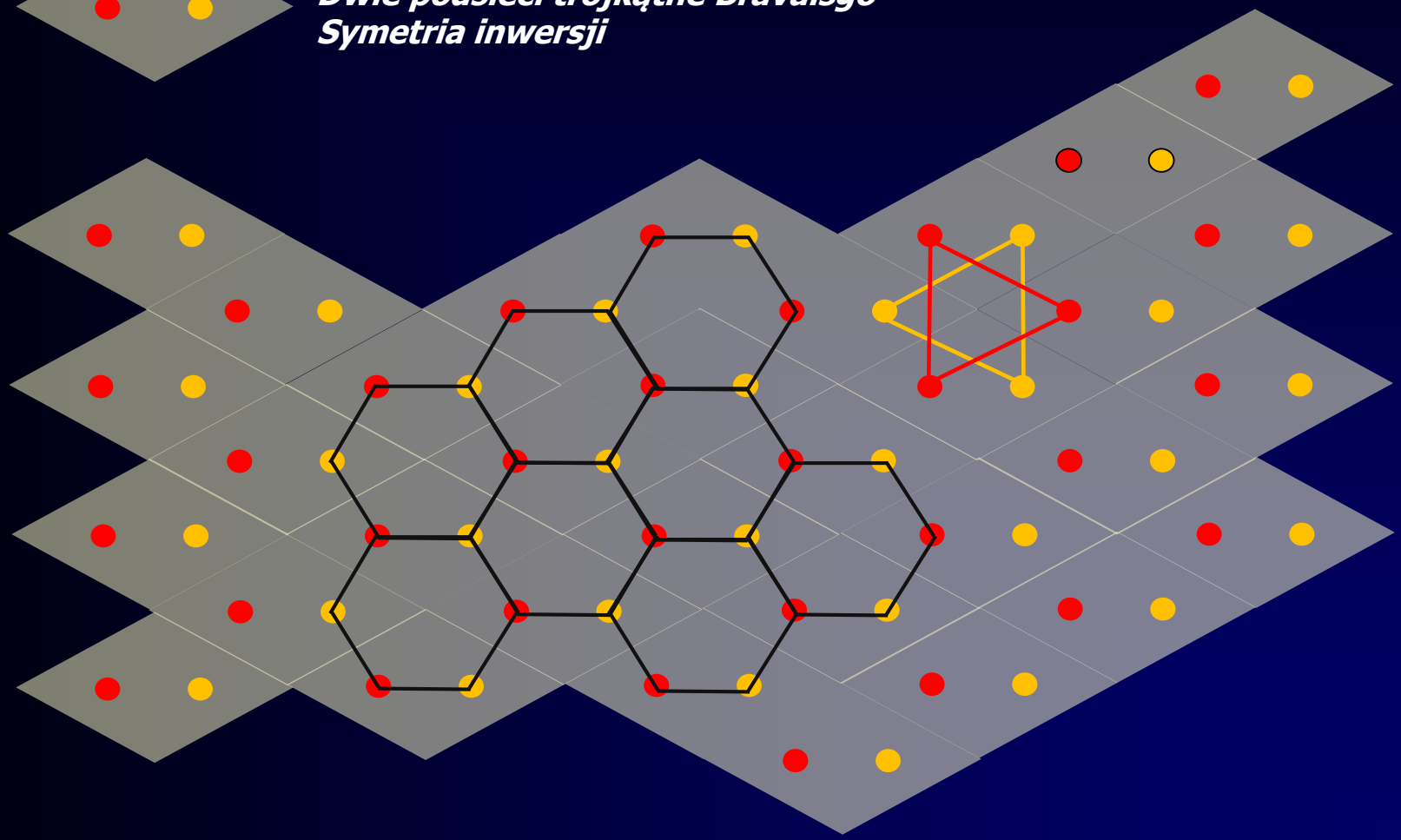
grafen



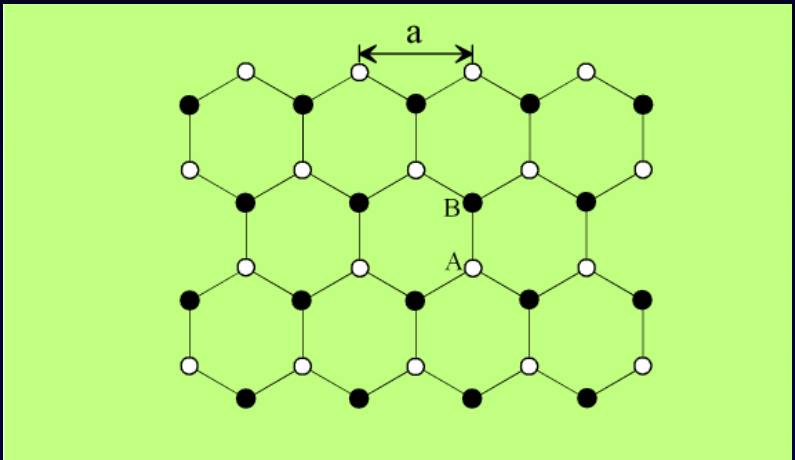
Grafen – sieć rzeczywista



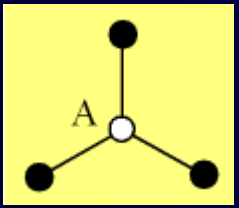
Komórka elementarna 2 atomy
Dwie podsieci trójkątne Bravaisgo
Symetria inwersji

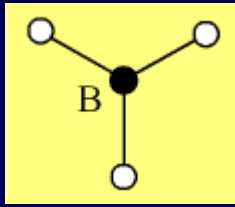


Grafen



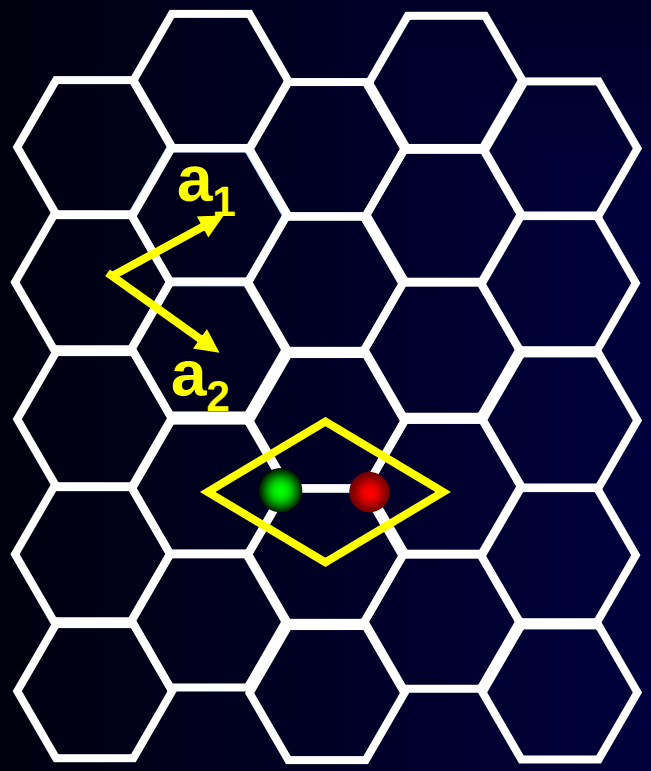
Potrzebne do koncepcji pseudospinu





Mamy dwie orientacje wiązań tzn. mamy dwa rodzaje atomów węgla.
Z chemicznego punktu widzenia nie ma to znaczenia.

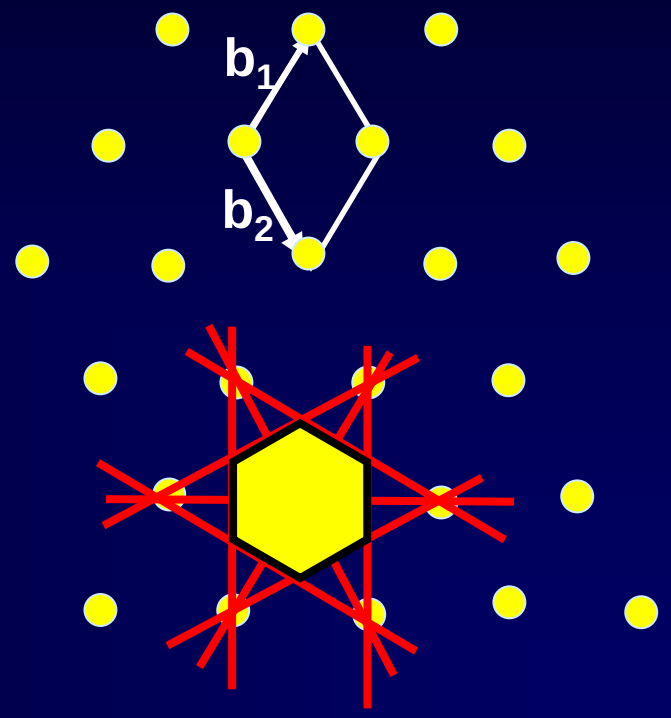
Grafen – sieć odwrotna



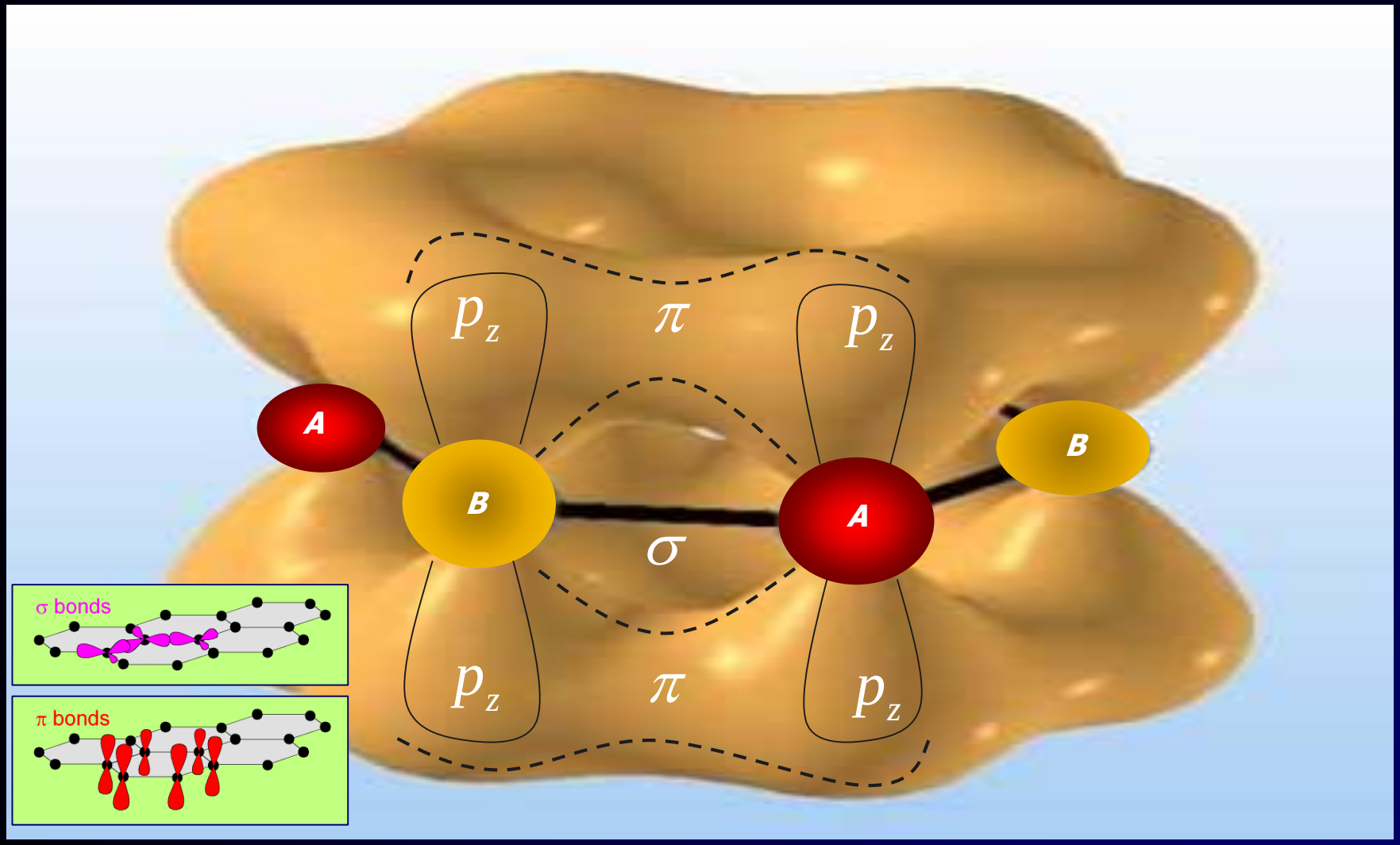
$$\vec{b}_1 \cdot \vec{a}_1 = 2\pi \qquad \vec{b}_1 \cdot \vec{a}_2 = 0$$

$$\vec{b}_2 \cdot \vec{a}_1 = 0 \qquad \vec{b}_2 \cdot \vec{a}_2 = 2\pi$$

Kmórka elementarna sieci odwrotnej
Konstrukcja Wignera-Seitza

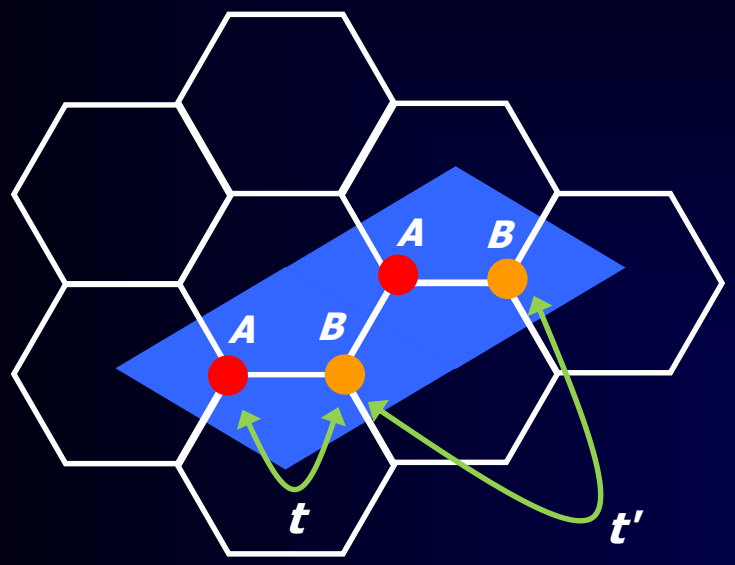


Grafen – struktura elektronowa

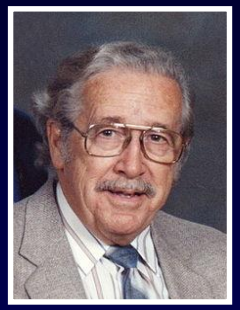


Grafen – teoria ciasnego wiązania

Jakie jest widmo energii kwazicząstek w grafenie ?



$$E\left(k_x, k_y\right) = f\left(k_x, k_y\right)$$



L.D. Landau

R.E. Peierls

Philip Russel Wallace, 1947

t=2.8 - 3 eV
przeskok pomiędzy dwoma różnymi podsieciami

t'=0.1eV przeskok w ramach tej samej sieci

$$\hat{H} = -t \sum_{<i,j>,\sigma} \left(a^{\dagger}_{\sigma,i} b_{\sigma,j} + h.c.\right) - t' \sum_{<<i,j>>,\sigma} \left(a^{\dagger}_{\sigma,i} a_{\sigma,j} + b^{\dagger}_{\sigma,i} b_{\sigma,j} + h.c.\right)$$

Grafen – teoria ciasnego wiązania

Jakie jest widmo energii kwazicząstek w grafenie ?

P.R. Wallace, Phys. Rev. 71 (1947) 622.

PHYSICAL REVIEW VOLUME 71, NUMBER 9 MAY 1, 1947

The Band Theory of Graphite

P. R. WALLACE*
National Research Council of Canada, Chalk River Laboratory, Chalk River, Ontario

(Received December 19, 1946)

The structure of the electronic energy bands and Brillouin zones for graphite is developed using the "tight binding" approximation. Graphite is found to be a semi-conductor with zero activation energy, i.e., there are no free electrons at zero temperature, but they are created at higher temperatures by excitation to a band contiguous to the highest one which is normally filled. The electrical conductivity is treated with assumptions about the mean free path. It is found to be about 100 times as great parallel to as across crystal planes. A large and anisotropic diamagnetic susceptibility is predicted for the conduction electrons; this is greatest for fields across the layers. The volume optical absorption is accounted for.

1. INTRODUCTION

THE purpose of this paper is to develop a basis for the explanation of some of the physical properties of graphite through the band theory of solids. We shall be concerned primarily with a discussion of its electrical conductivity, but the treatment given makes possible the explanation not only of the electrical conductivity and its anisotropy but also the thermal conductivity, diamagnetic susceptibility, and optical absorption.

The electrical resistivity of single crystals of graphite is about 4 to 6×10^{-6} ohm-cm.[1] This corresponds to a conductivity of the order of that of a poor metal. The temperature coefficient of the conductivity is negative, as in the case of a metal. Polycrystalline graphite, on the other hand, has a much higher resistivity which varies very strongly according to the type of graphite used, and has a *positive* temperature coefficient of conductivity[2] to about 1400°C, and negative thereafter. Since the crystals of commercial graphites tend to be of the order of 10^{-6} cm, and it is quite porous (density ~1.6 as against 2.25 for single crystals), it seems reasonable to attribute the high resistivity of polycrystalline graphite to the crystal boundaries, on which may be lodged impurity atoms. The latter would tend to be driven off on heating, thus accounting for the observed temperature dependence. We shall show, however, that the band theory would seem to make possible the explanation of the conductivity properties of single crystals.

* Now at McGill University.
[1] Given by E. Ryschewitsch, Zeits. f. Elektrochem. ang. physik. Chemie **29**, 474 (1923), as $3.9\text{–}6\times10^{-6}$ ohm-cm.

[2] C. A. Hansen, Trans. Am. Electrochem. Soc. 16, 329 (1909) gives 137.5×10^{-6} at 0°C 82.5×10^{-6} at 1400°C.

Single hexagonal layer

Graphene -1986

Carbon Vol. 24, No. 2, pp. 241–245, 1986
Printed in Great Britain.
0008–6223/86 $3.00+.00
1986 Pergamon Press Ltd.

EDITORIAL SECTION

NOMENCLATURE AND TERMINOLOGY OF GRAPHITE INTERCALATION COMPOUNDS

H. P. BOEHM
Institut für Anorganische Chemie der Universität München, Meiserstraße 1, D-8000 München 2, F.R.G.

The ending *-ene* is used for fused polycyclic aromatic hydrocarbons, even when the root of the name is of trivial origin, e.g. naphthalene, anthracene, tetracene, coronene, ovalene. A single carbon layer of the graphitic structure would be the final member of infinite size of this series. The term *graphene* layer should be used for such a single carbon layer.‡

Grafen – teoria ciasnego wiązania

Jakie jest widmo energii kwazicząstek w grafenie ?

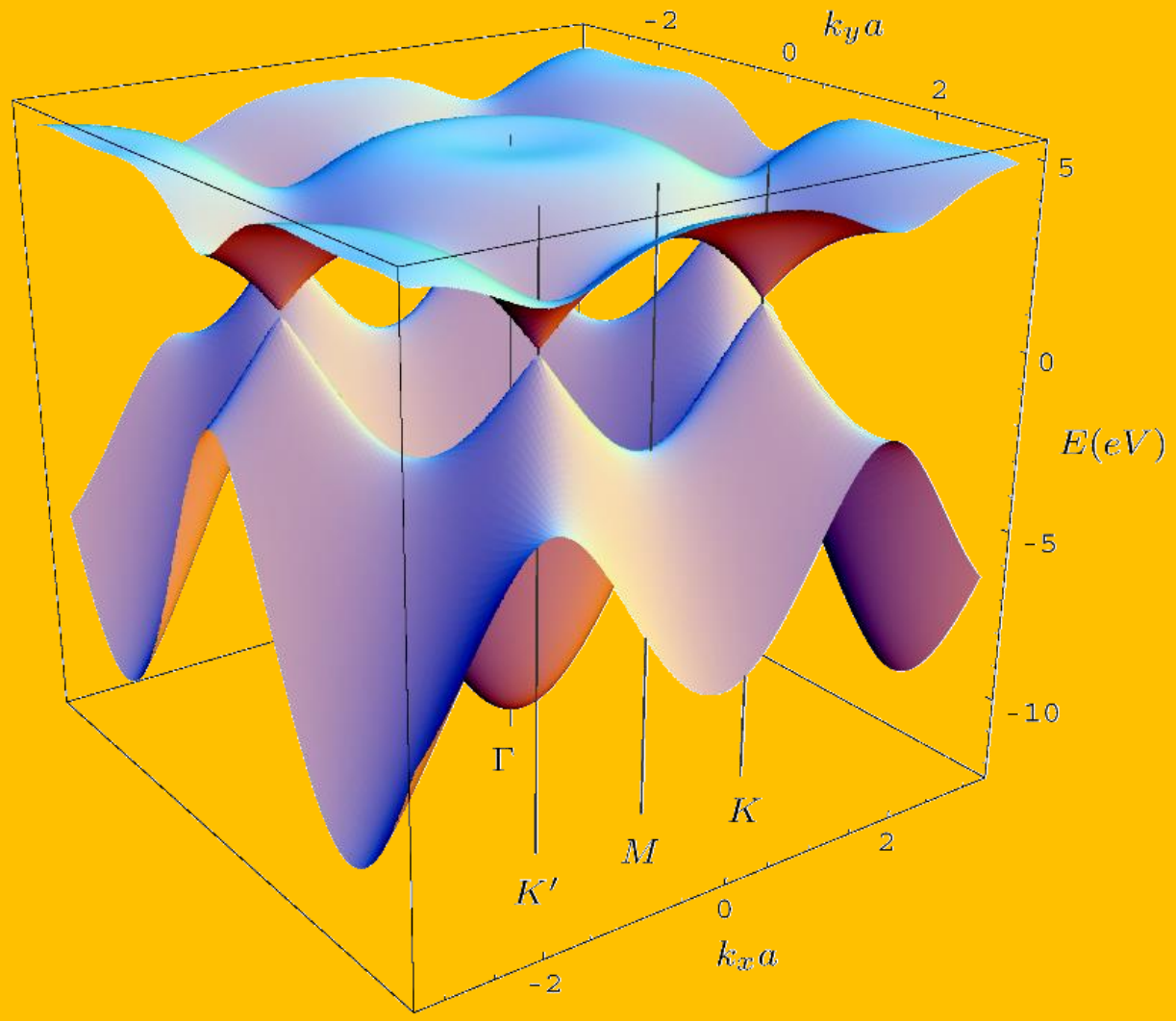
$$\hat{H} = -t \sum_{<i,j>,\sigma} \left(a^{\dagger}_{\sigma,i} b_{\sigma,j} + h.c. \right) - t' \sum_{<<i,j>>,\sigma} \left(a^{\dagger}_{\sigma,i} a_{\sigma,j} + b^{\dagger}_{\sigma,i} b_{\sigma,j} + h.c. \right)$$

$$E\left(k_x, k_y\right) = E\left(\vec{k}\right) = \pm t \sqrt{3 + f\left(\vec{k}\right)} - t' f\left(\vec{k}\right)$$

$$f\left(\vec{k}\right) = 2\cos\left(\sqrt{3} a k_y\right) + 4\cos\left(\frac{\sqrt{3} a k_y}{2}\right)\cos\left(\frac{3}{2} k_x x\right)$$

Grafen – teoria ciasnego wiązania

Jakie jest widmo energii kwazicząstek w grafenie ?



Grafen – teoria ciasnego wiązania

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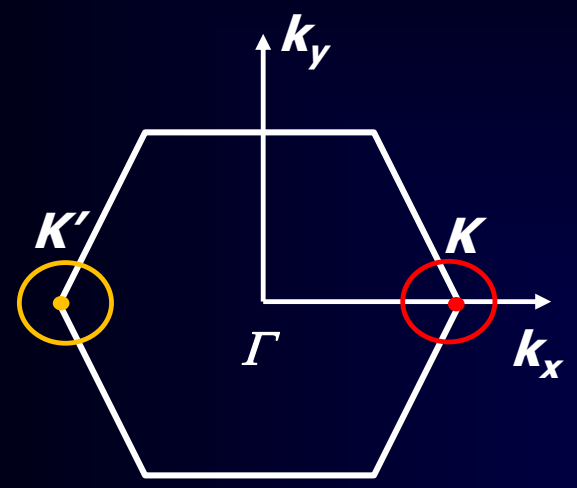
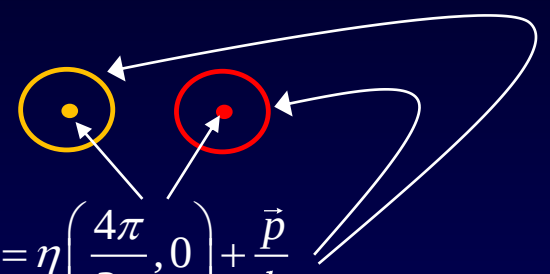


$$\vec{K} = \left(\frac{4\pi}{3a}, 0\right); \quad \vec{K}' = \left(-\frac{4\pi}{3a}, 0\right)$$

$$\vec{K}, \vec{K}' = \eta\left(\frac{4\pi}{3a}, 0\right); \quad \eta = \pm 1$$

Rozwijamy funkcję f(k) w otoczeniu punktów K i K'



$$\vec{k} = \eta\left(\frac{4\pi}{3a}, 0\right) + \frac{\vec{p}}{\hbar}$$

$$f\left(\vec{k}\right) = e^{ik_y a/\sqrt{3}} + 2e^{-ik_y a/2\sqrt{3}} \cos\left(\frac{k_x a}{2}\right)$$

$$f\left(\vec{k}\right) = -\frac{\sqrt{3}a}{2\hbar}\left(\eta p_x - i p_y\right) + O\left(pa/\hbar\right)^2$$

Grafen – teoria ciasnego wiązania

Jakie jest widmo energii kwazicząstek w grafenie ?

$$H = \begin{pmatrix} 0 & -tf\left(\vec{k}\right) \\ -tf^*\left(\vec{k}\right) & 0 \end{pmatrix} \approx v \begin{pmatrix} 0 & p_x - ip_y \\ p_x + ip_y & 0 \end{pmatrix}$$

$$S = \begin{pmatrix} 1 & sf\left(\vec{k}\right) \\ sf^*\left(\vec{k}\right) & 1 \end{pmatrix} \approx \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$H = v \begin{pmatrix} 0 & p_x - ip_y \\ p_x + ip_y & 0 \end{pmatrix} = v\left(\sigma_x p_x + \sigma_y p_y\right) = v\vec{\sigma}\vec{p}$$

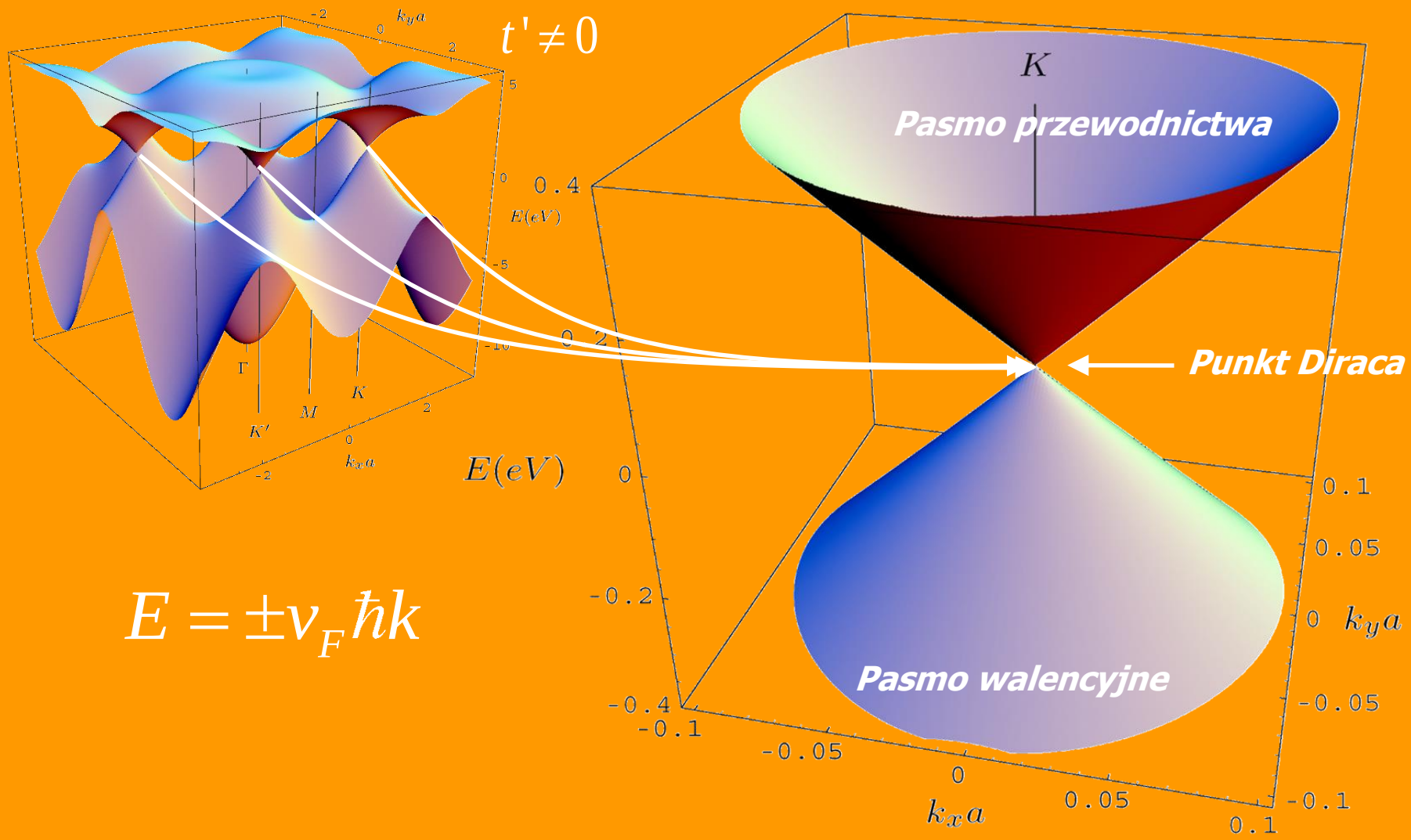
W. Pauli

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$E = \pm v\hbar k$$

Grafen – teoria ciasnego wiązania

Jakie jest widmo energii kwazicząstek w grafenie ?

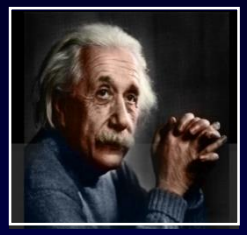


$$E = \pm v_F \hbar k$$

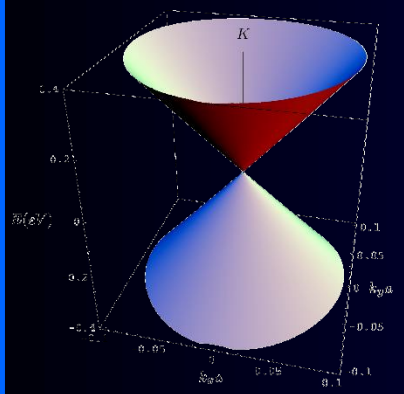
Grafen - struktura pasmowa

Relatywistyczna zależność pomiędzy energią, pędem i masą spoczynkową

$$E = \pm\sqrt{m_0^2 c^4 + p^2 c^2}$$



A. Einstein



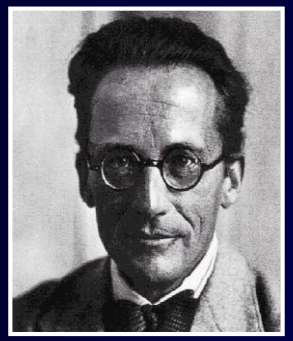
Przypadek cząstek z zerową masą spoczynkową

$$m_0 = 0 \qquad E = \pm cp = \pm c\hbar k$$

Grafen - struktura pasmowa

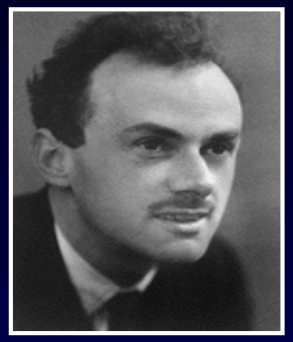
$$-\frac{\hbar^2}{2m_e^*}\Delta\psi + \hat{V}\psi = E\psi$$

$$-\frac{\hbar^2}{2m_h^*}\Delta\psi + \hat{V}\psi = E\psi$$



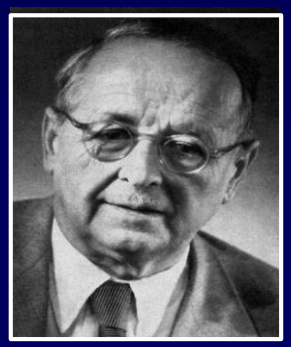
E. Schrodinger

fermiony z masą



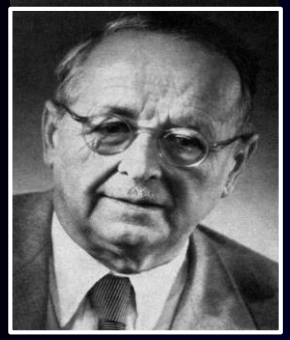
P. A. M. Dirac

bezmasowe fermiony



H. Weyl

Grafen - struktura pasmowa



H. Weyl

$$H_K = -\frac{\sqrt{3}}{2} at\left(\sigma_x k_x + \sigma_y k_y\right) = -\frac{\sqrt{3}}{2} at\vec{\sigma}\cdot\vec{k}$$

$$H_K = v_F \hbar\left(\sigma_x k_x + \sigma_y k_y\right) = v_F \hbar\vec{\sigma}\cdot\vec{k}$$

$$v_F \equiv c$$

W. Pauli

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Masa efektywna

$$\left(\frac{1}{m^*}\right)_{i,j} = \frac{1}{\hbar^2}\frac{d^2E(k)}{dk_i dk_j}$$

i,j odpowiadają kierunkom x,y,z

$$\left(\frac{1}{m^*}\right) = \begin{pmatrix} \frac{1}{\hbar^2}\frac{\partial^2 E(k)}{\partial k_x^2} & \frac{1}{\hbar^2}\frac{\partial^2 E(k)}{\partial k_x \partial k_y} & \frac{1}{\hbar^2}\frac{\partial^2 E(k)}{\partial k_x \partial k_z} \\ \frac{1}{\hbar^2}\frac{\partial^2 E(k)}{\partial k_y \partial k_x} & \frac{1}{\hbar^2}\frac{\partial^2 E(k)}{\partial k_y^2} & \frac{1}{\hbar^2}\frac{\partial^2 E(k)}{\partial k_y \partial k_z} \\ \frac{1}{\hbar^2}\frac{\partial^2 E(k)}{\partial k_z \partial k_x} & \frac{1}{\hbar^2}\frac{\partial^2 E(k)}{\partial k_z \partial k_y} & \frac{1}{\hbar^2}\frac{\partial^2 E(k)}{\partial k_z^2} \end{pmatrix}$$

Masa efektywna

$$m^* = \hbar^2 \frac{1}{\frac{\partial^2 E}{\partial k^2}}$$

$$m^* = \hbar^2 k \frac{1}{\frac{\partial E}{\partial k}}$$

Masa efektywna dla liniowej zależności dyspersyjnej

Masa efektywna dla dowolnej zależności dyspersyjnej

$$m^* = \hbar^2 \frac{1}{\frac{\partial^2 E}{\partial k^2}} \Rightarrow \infty$$

$$m^* = \hbar^2 k \frac{1}{\frac{\partial E}{\partial k}} = \frac{\hbar k}{v_F}$$

Trochę eksperymentu

Grafen - struktura pasmowa

$$H_K = -\frac{\sqrt{3}}{2} at \left[k_x \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + k_y \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \right]$$

$$H_K = \begin{pmatrix} 0 & -\frac{\sqrt{3}}{2} at \left(k_x - ik_y \right) \\ -\frac{\sqrt{3}}{2} at \left(k_x + ik_y \right) & 0 \end{pmatrix}$$

$$\det\left(H_K - EI \right) = 0$$

równanie wiekowe
równanie sekularne

$$\det\left(H_K - EI \right) = \det \left[\begin{pmatrix} 0 & -\frac{\sqrt{3}}{2} at \left(k_x - ik_y \right) \\ -\frac{\sqrt{3}}{2} at \left(k_x + ik_y \right) & 0 \end{pmatrix} - E \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] = 0$$

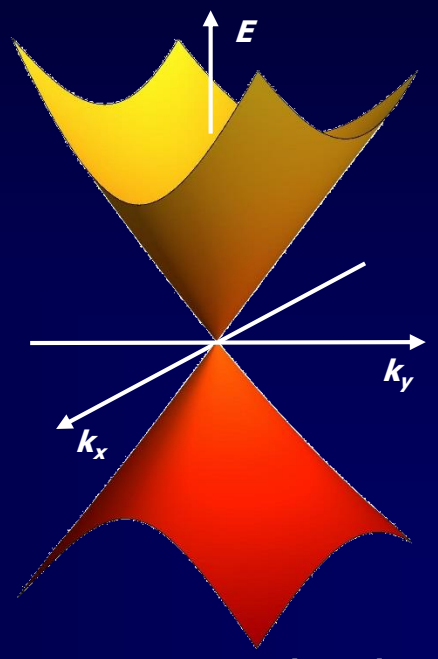
Grafen - struktura pasmowa

$$\det\left(H_K - EI\right) = E^2 - \frac{3}{4}(at)^2\left(k_x^2 + k_y^2\right) = 0$$

$$E^2 = \frac{3}{4}(at)^2\left(k_x^2 + k_y^2\right)$$

$$E = \pm\frac{\sqrt{3}}{2}at\sqrt{\left(k_x^2 + k_y^2\right)}$$

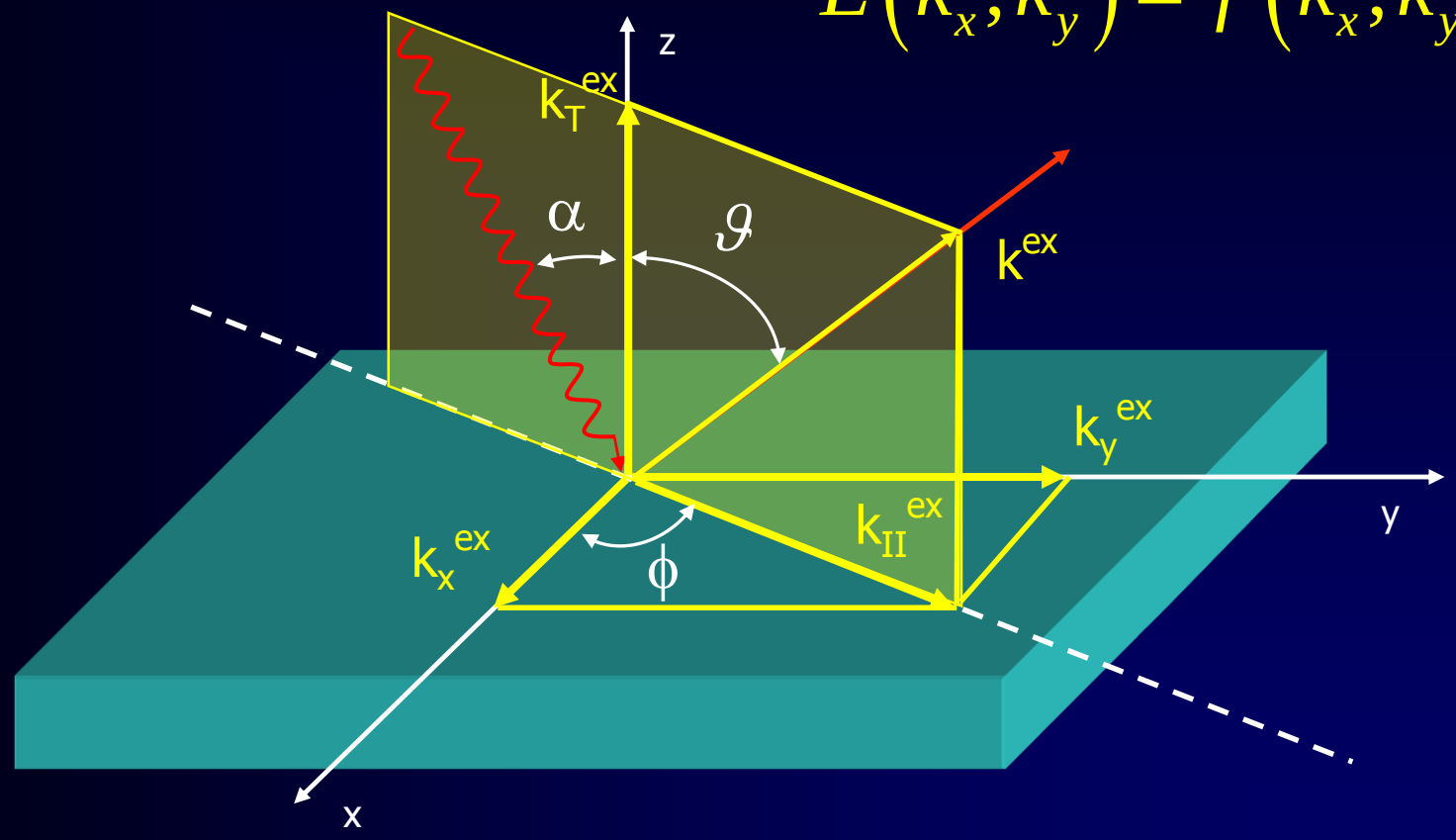
$$E = \pm v_F \hbar \ |k|$$



Bezmasowe Fermiony Diraca

ARPES – kątowo rozdzielcza fotoemisja elektronów

$$E\left(k_x, k_y\right) = f\left(k_x, k_y\right)$$



ARPES – kątowo rozdzielcza fotoemisja elektronów

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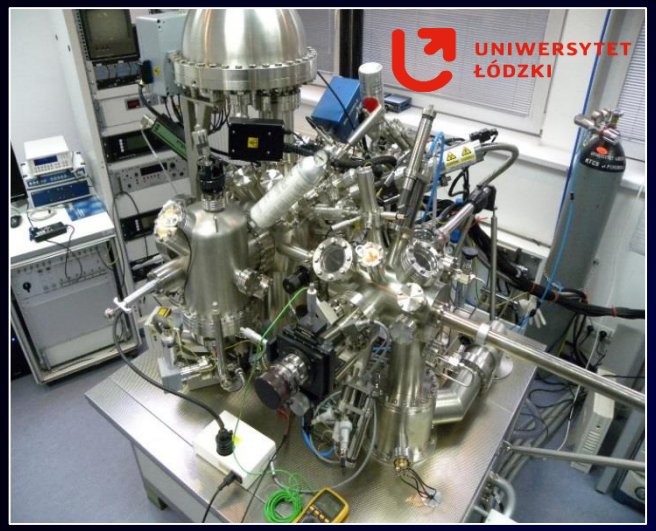

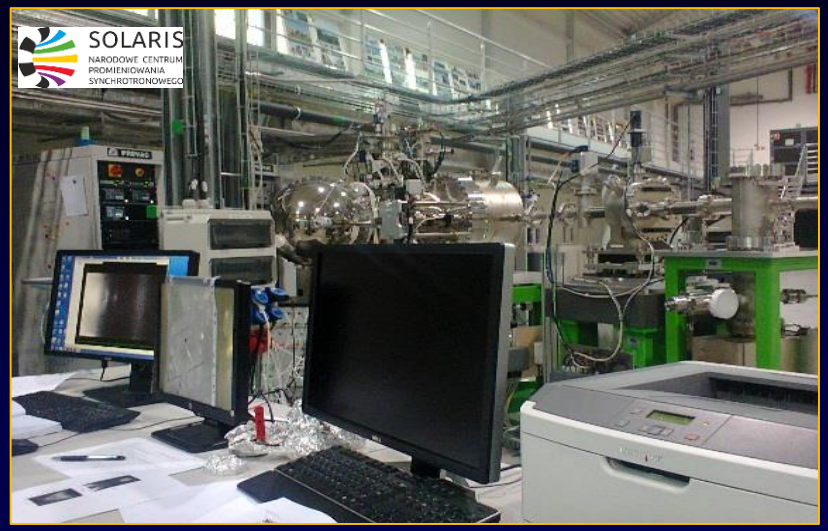

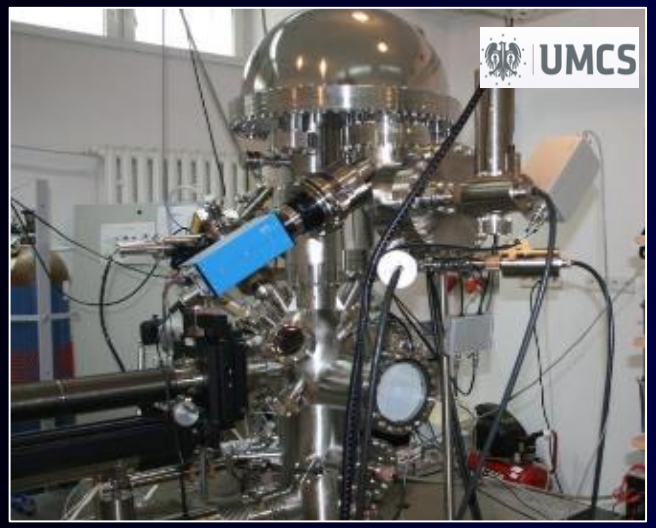

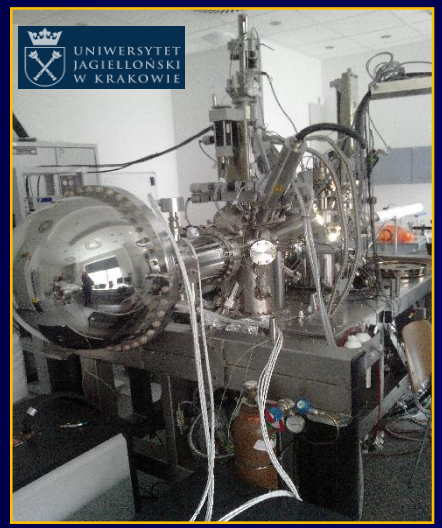

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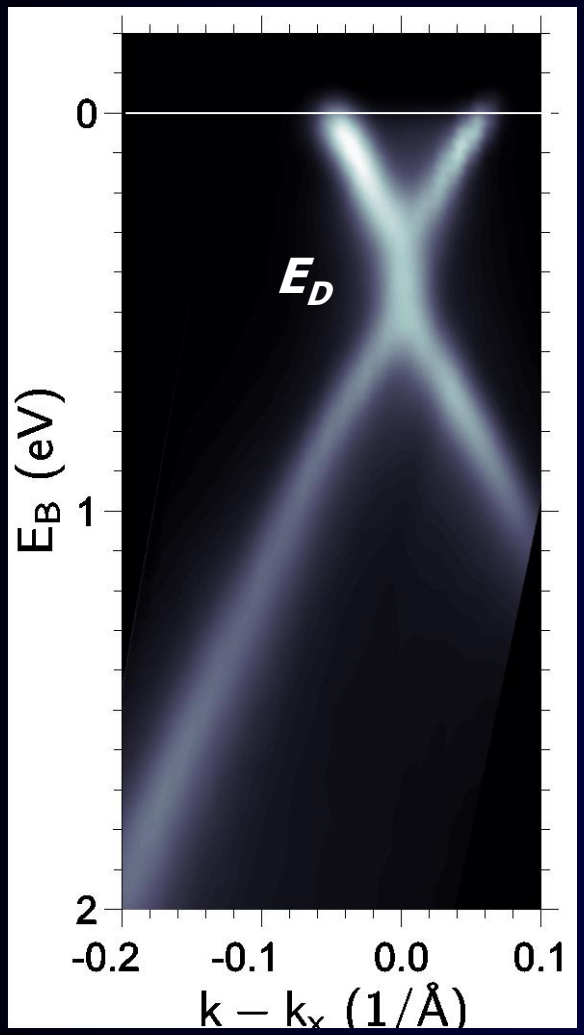
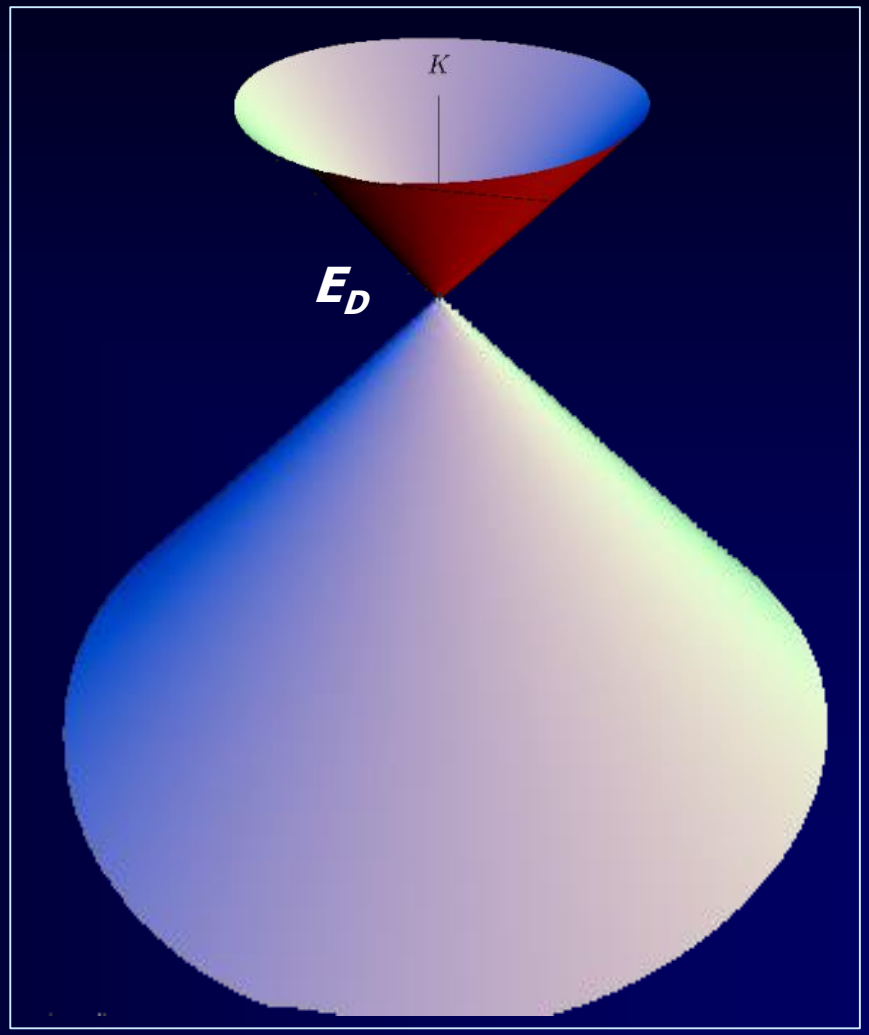
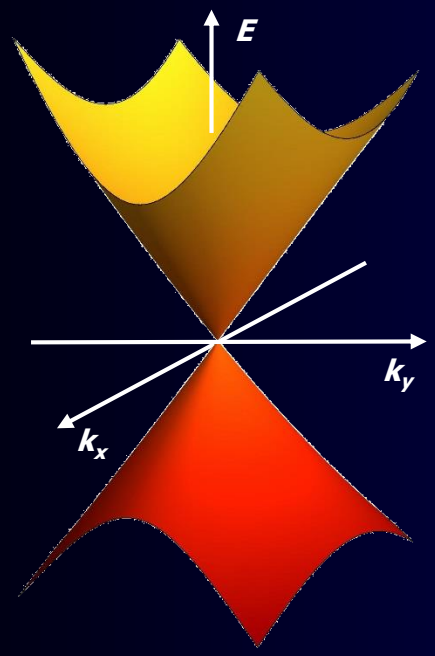




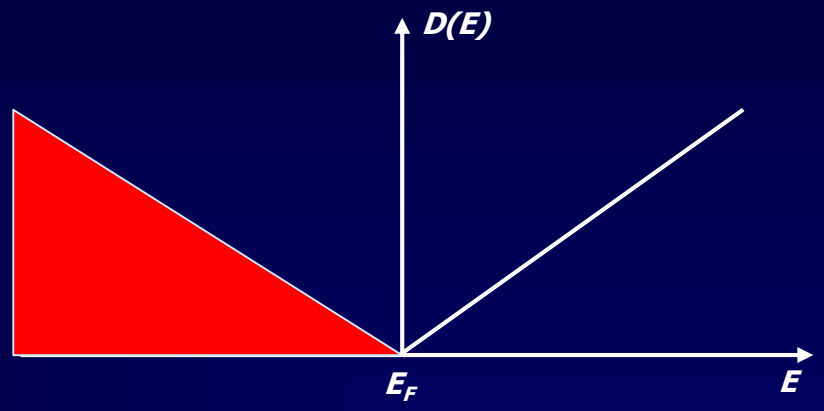
Grafen - struktura pasmowa

E(k)->D(E)

Zależność dyspersyjna Funkcja gęstości stanów



Bezmasowe Fermiony Diraca



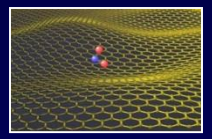
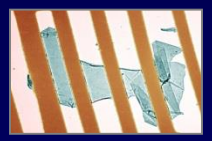
Funkcja gęstości stanów

$$\sigma \approx en\mu$$

Grafen - struktura pasmowa

Ruchliwość nośników
Przewodnictwo elektryczne

$$\mu = \frac{v}{E} \qquad E = \frac{U}{d}\left[\frac{V}{cm}\right] \qquad v = \frac{d}{t}\left[\frac{cm}{s}\right] \Rightarrow \sigma \approx en\mu$$

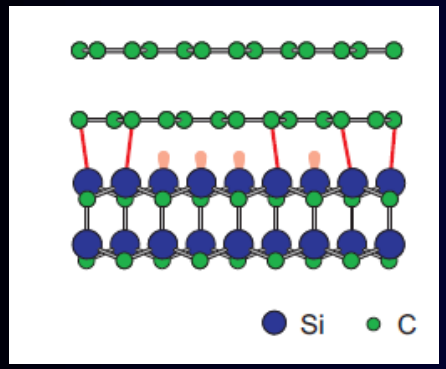
$10\,000 - 15\,000\, cm^2V^{-1}s^{-1}$	**Grafen na SiO_2**	
$200\,000\, cm^2V^{-1}s^{-1}$	**Grafen na SiO_2 bez zanieczyszczeń**	
$1\,000\,000\, cm^2V^{-1}s^{-1}$	**Grafen zawieszony**	

T=300 K
Bez domieszkowania

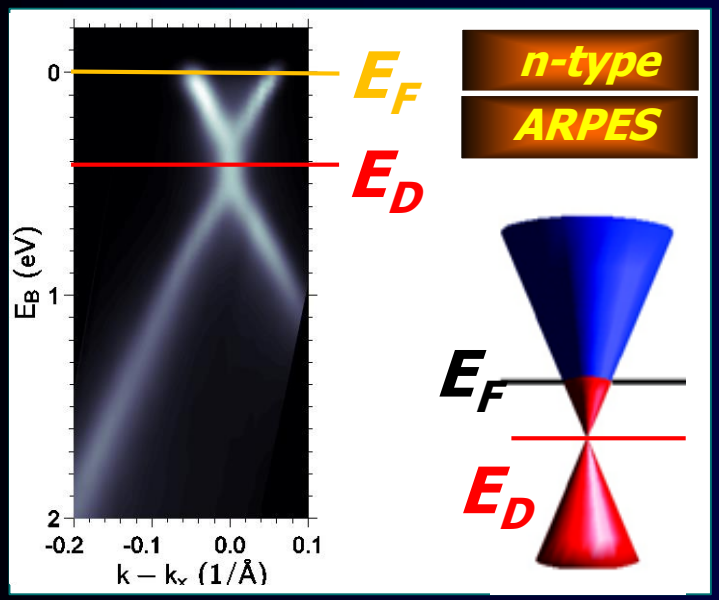
Si	$1\,300$	$cm^2V^{-1}s^{-1}$	$GaAs$	$8\,800$	$cm^2V^{-1}s^{-1}$
Ge	$3\,800$	$cm^2V^{-1}s^{-1}$	$InSb$	$78\,000$	$cm^2V^{-1}s^{-1}$

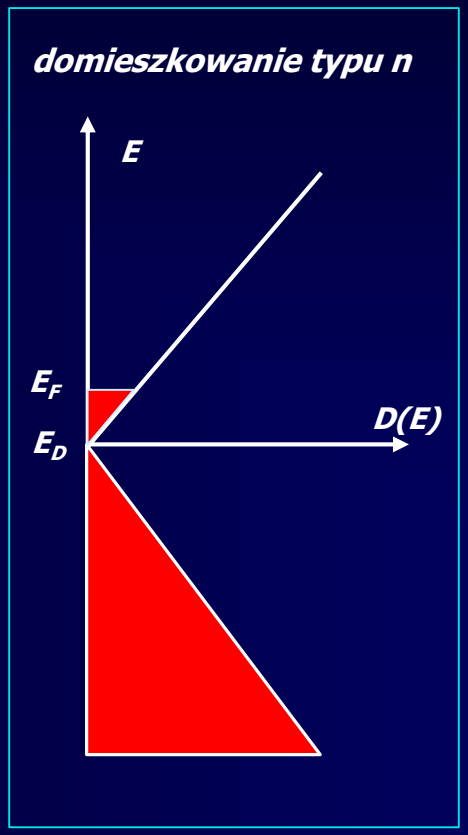
grafen/4H-SiC(0001) – $H_{interkalacja}$

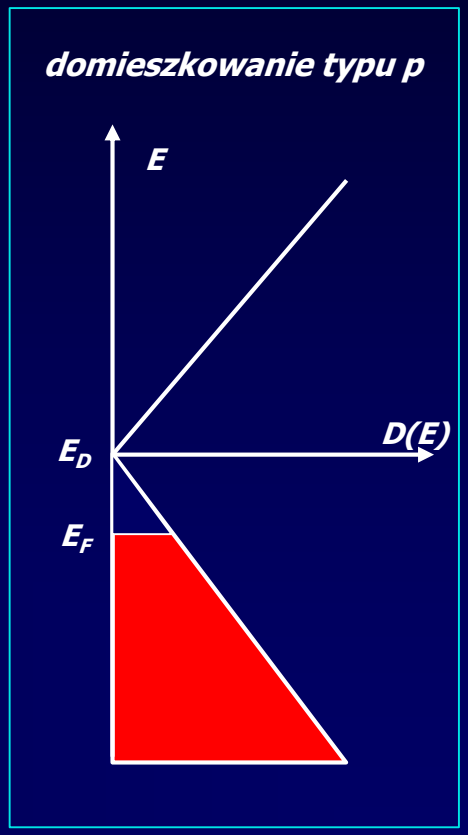
grafen/SiC



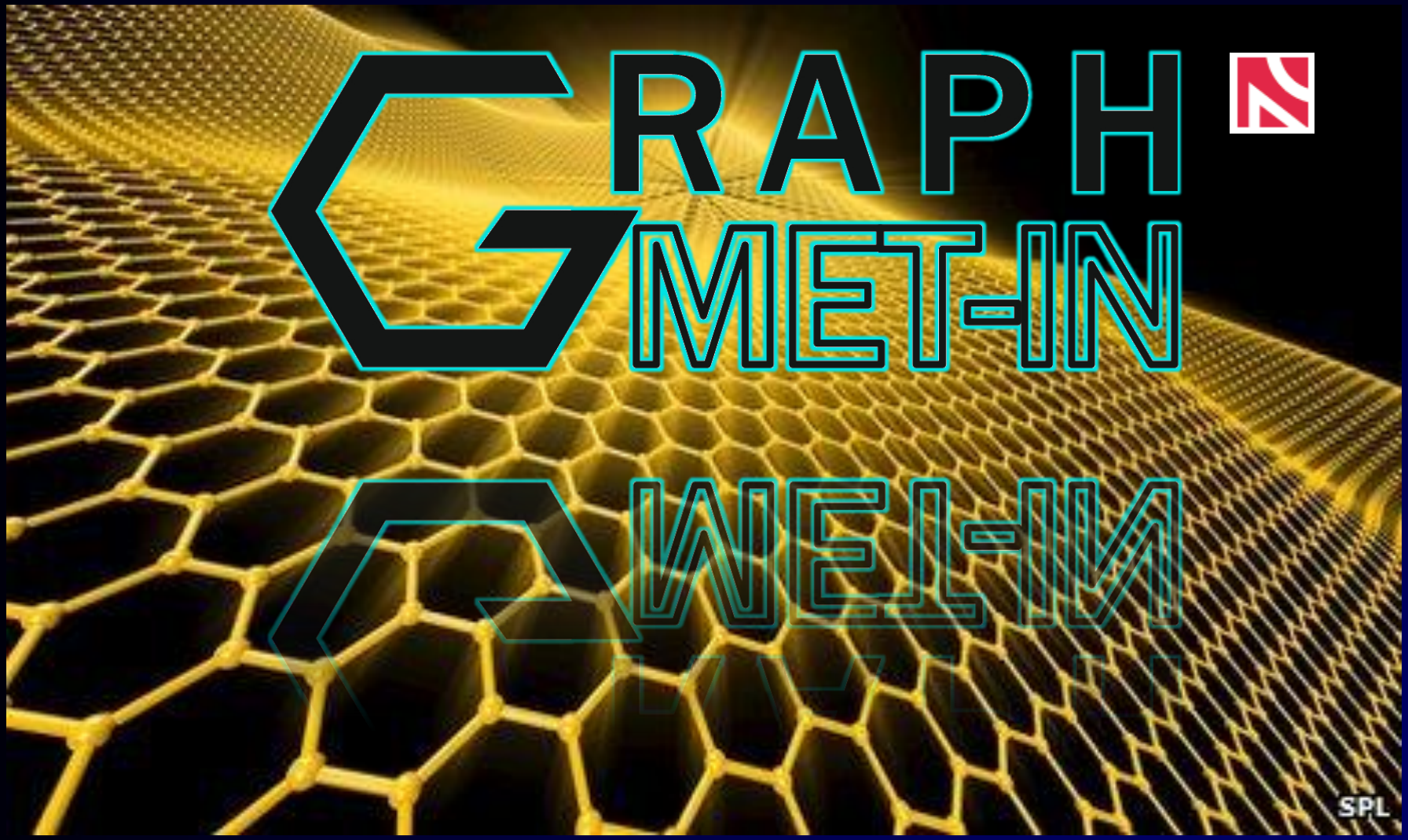
J.M. Baranowski, M. Możdżonek, P. Dąbrowski,
K. Grodecki, P. Osewski, W. Kozłowski,
M. Kopciuszyński, M. Jałochowski, W. Strupiński
Graphene, 2, 115120 (2013)







Oddziaływanie grafenu z metalami i izolatorami



Z. Klusek

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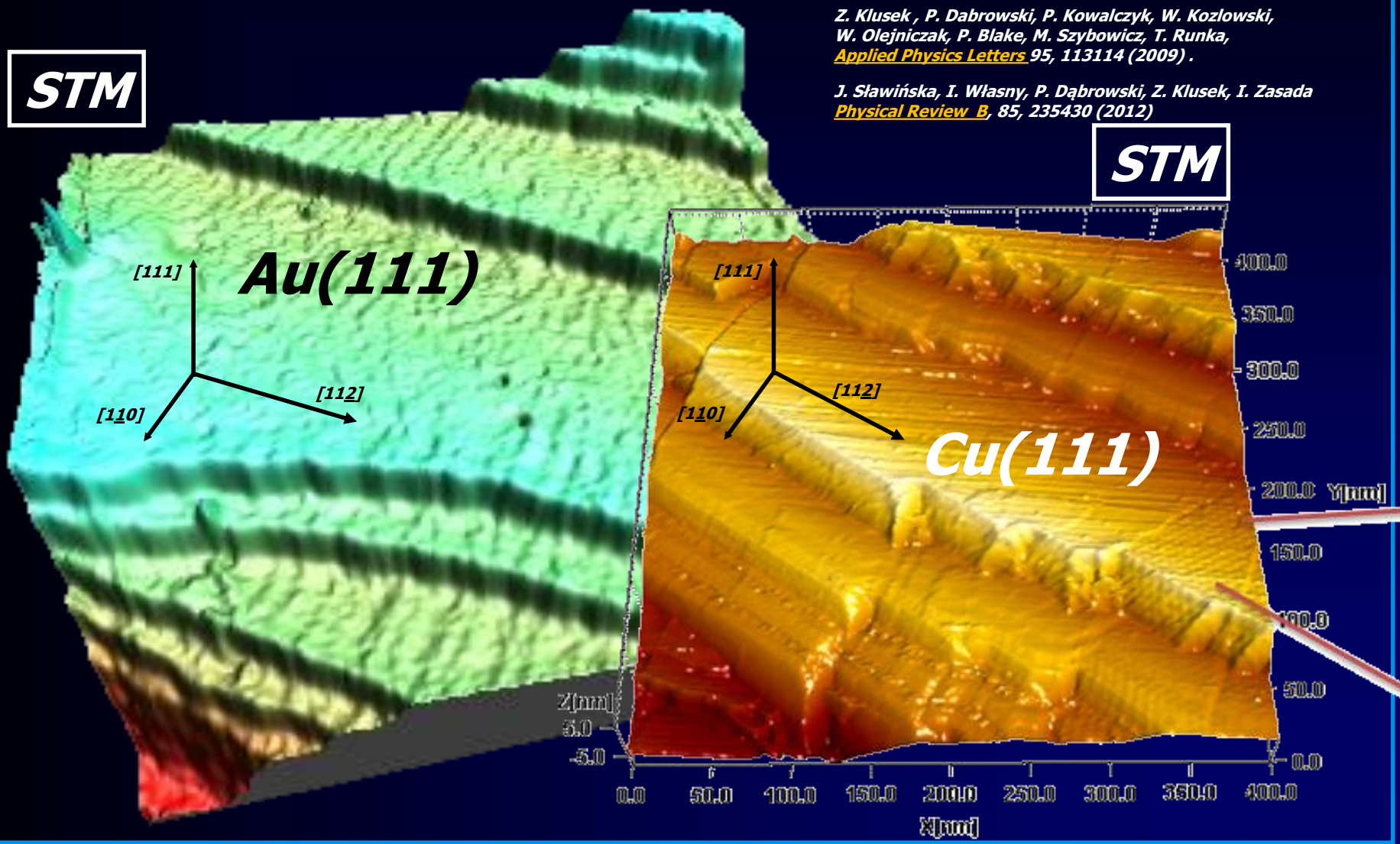
Oddziaływanie grafenu z metalami

Z. Klusek , P. Dabrowski, P. Kowalczyk, W. Kozlowski, W. Olejniczak, P. Blake, M. Szybowicz, T. Runka, Applied Physics Letters 95, 113114 (2009) .

J. Sławińska, I. Własny, P. Dąbrowski, Z. Klusek, I. Zasada Physical Review B, 85, 235430 (2012)



DFT: Oddziaływanie grafen-metal

Własności elektronowe grafenu ulegają zmianie na skutek oddziaływania z metalami

Oddziaływanie grafen-metal silne/słabe

chemisorpcja	fizysorpcja
Co(111) Ni(111) Pd(111)	Al(111) Ag(111) Cu(111) Au(111) Pt(111)

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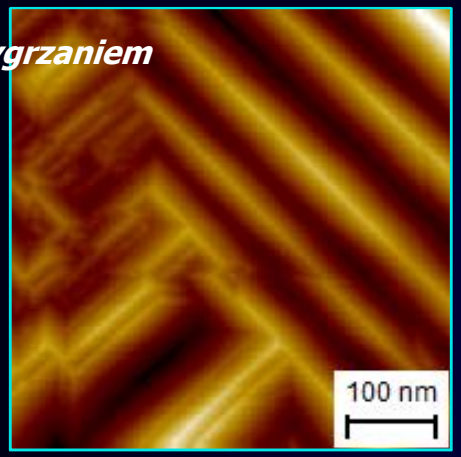
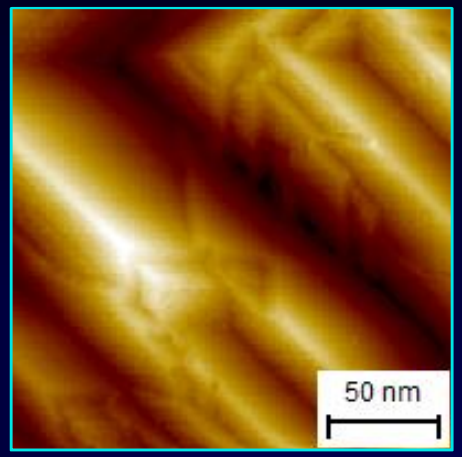
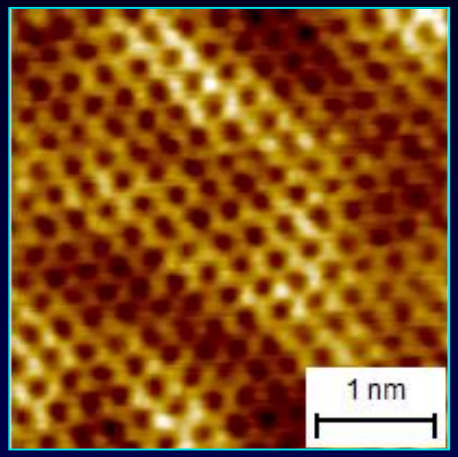
grafen/Ge(001)/Si

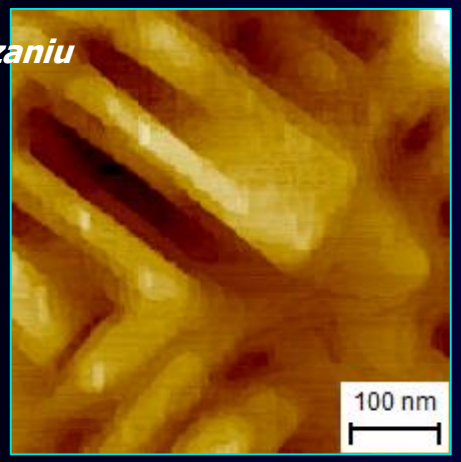
Przed wygrzaniem

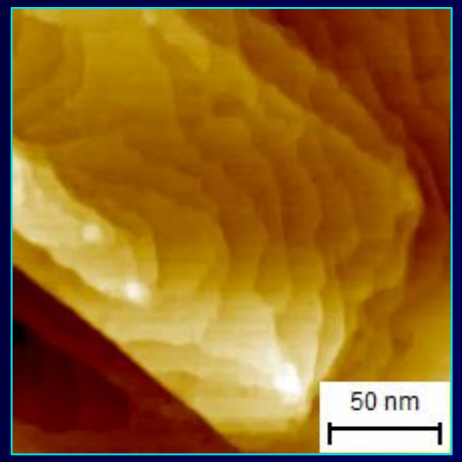


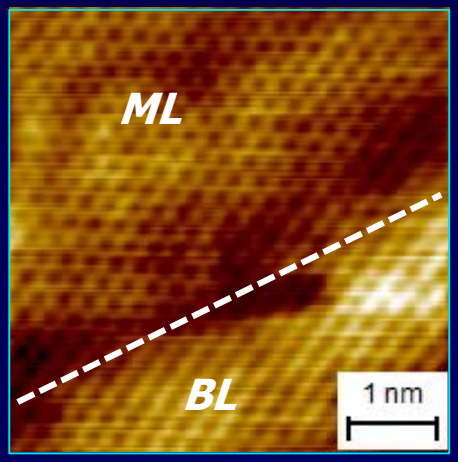




Po wygrzaniu







P. Dabrowski, M. Rogala, I. Pasternak, J. M. Baranowski, W. Strupinski, M. Kopciuszynski, R. Zdyb, M. Jalochowski, I. Lutsyk, Z. Klusek
Nano Res. doi:10.1007/s12274-017-1575-6 First Online:06 May 2017

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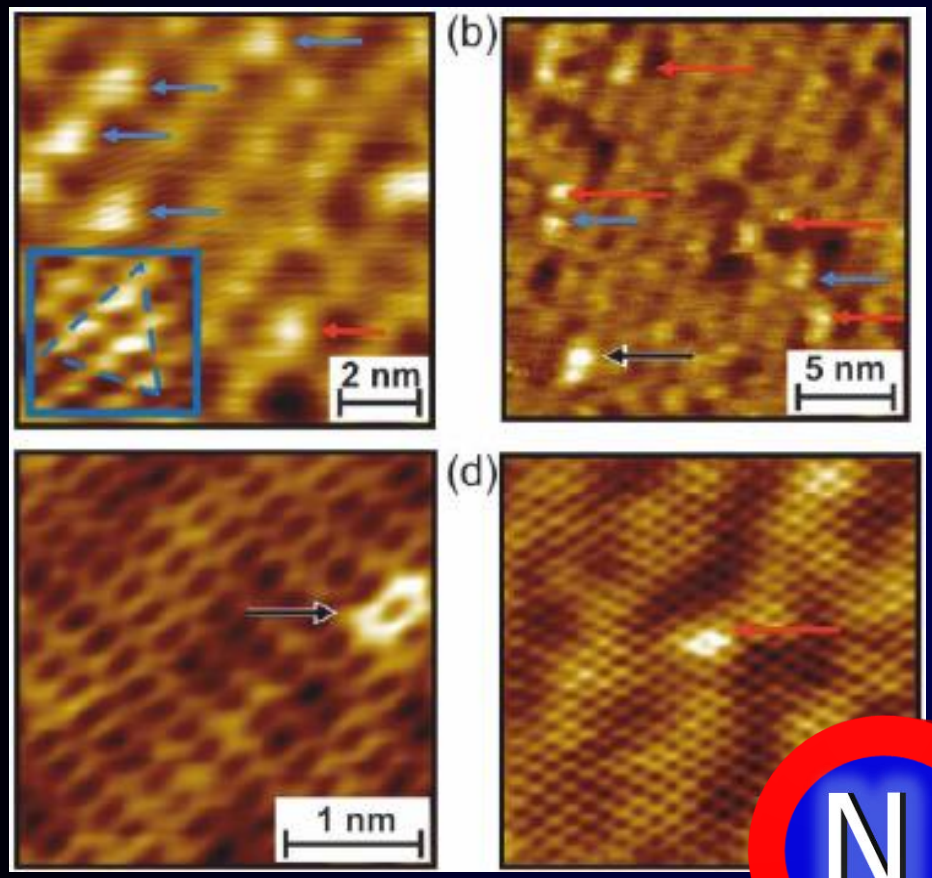
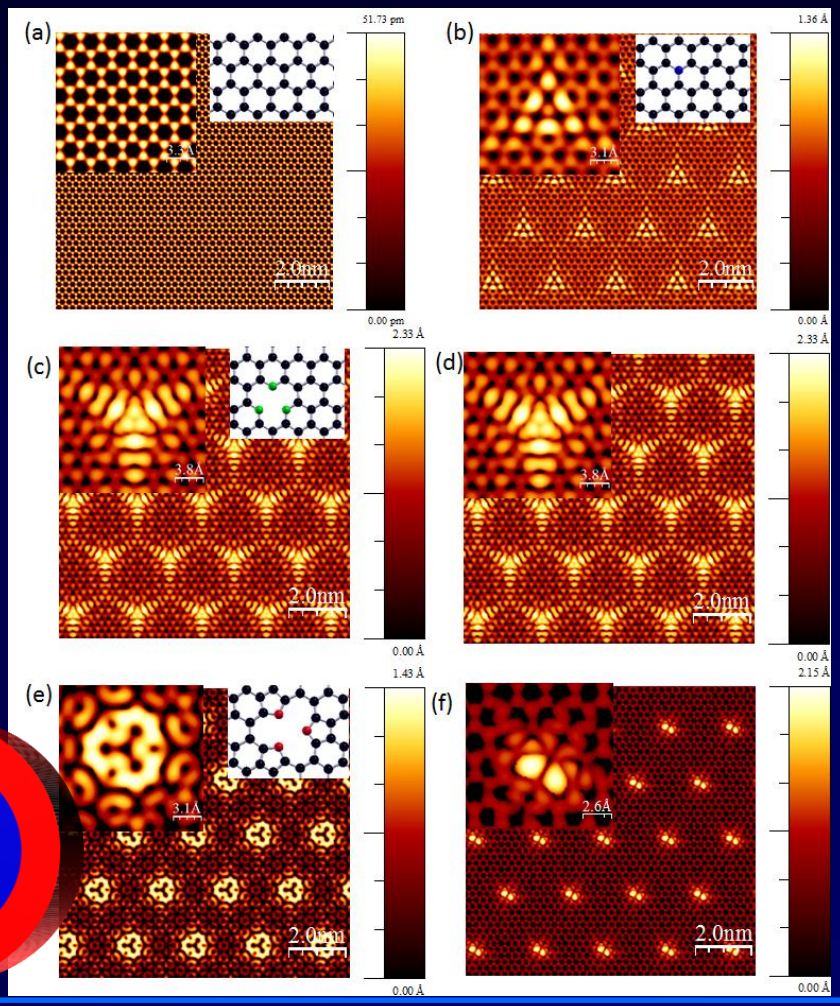
grafen/4H-SiC(0001) – N doped

STM

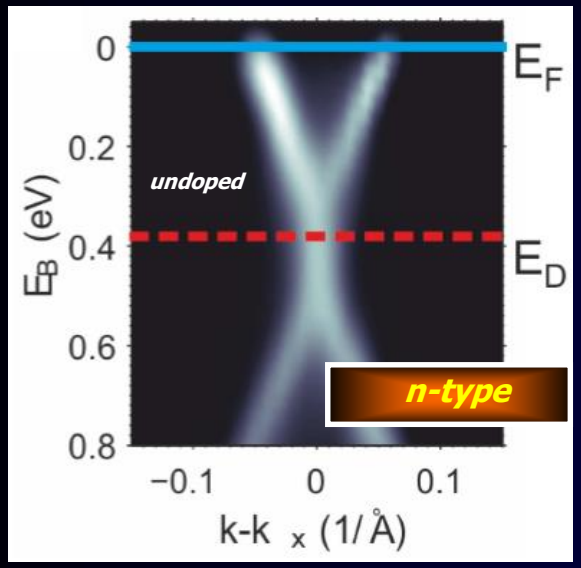


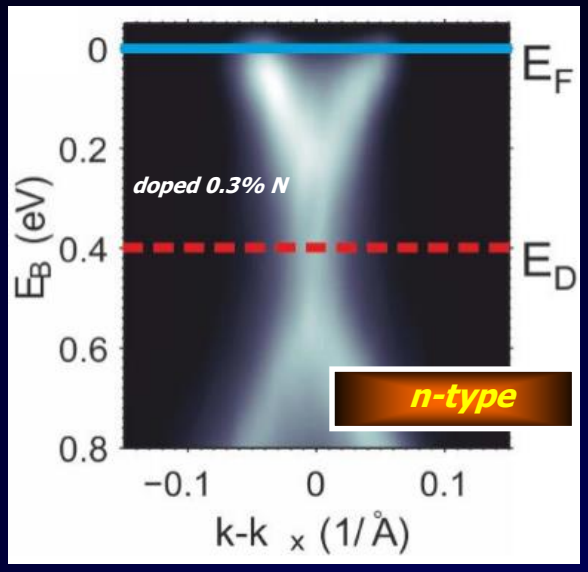
DFT

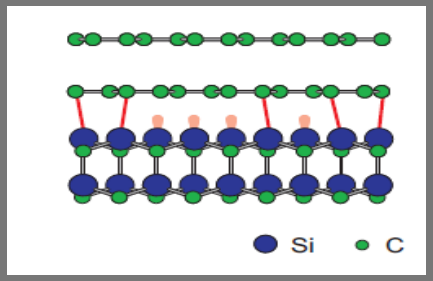


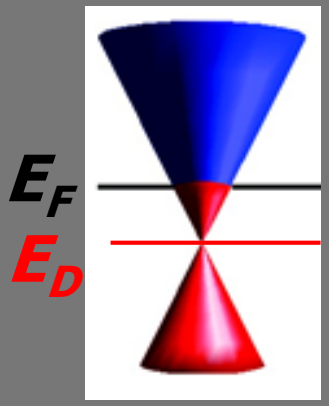
P. Dąbrowski, M. Rogala, I. Własny, Z. Klusek, M. Kopciuszyński, M. Jałochowski, W. Strupiński, J.M. Baranowski
Carbon 94, 214-223 (2015)

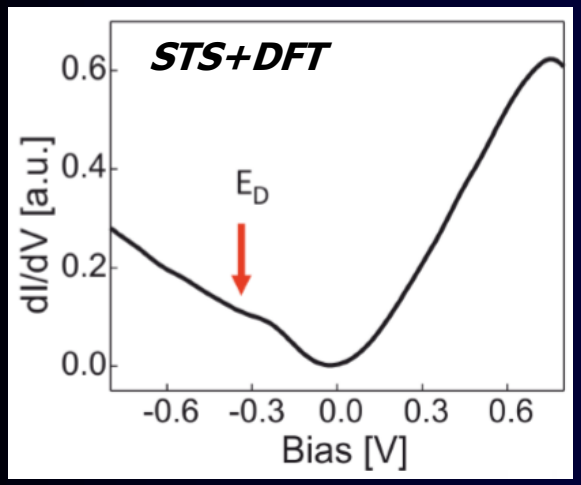
grafen/4H-SiC(0001) – N doped

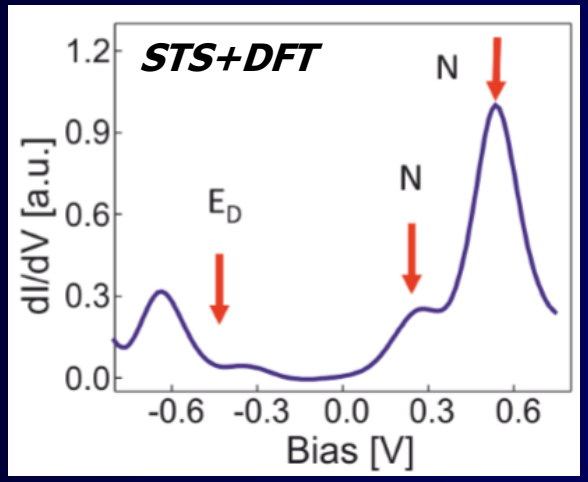






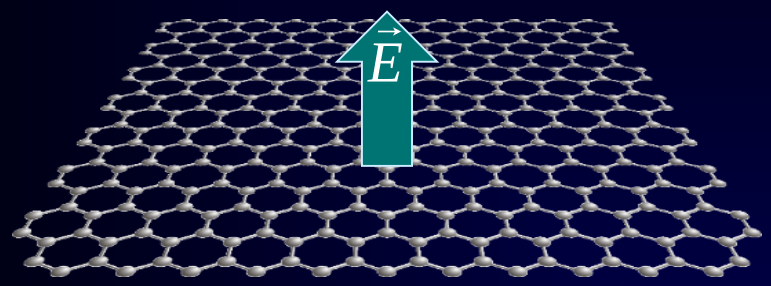


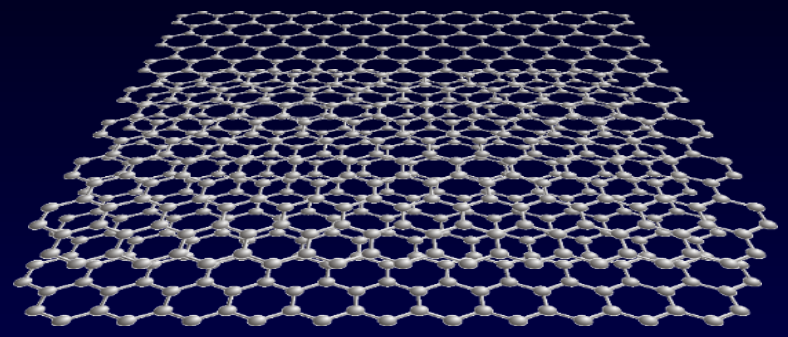




P. Dąbrowski, M. Rogala, I. Własny, Z. Klusek, M. Kopciuszyński, M. Jałochowski, W. Strupiński, J.M. Baranowski
Carbon 94, 214-223 (2015)

Grafen – w zewnętrznym potencjale





$$H_K = -\frac{\sqrt{3}}{2} at\left(\sigma_x k_x + \sigma_y k_y\right) = -\frac{\sqrt{3}}{2} at\vec{\sigma}\cdot\vec{k}$$

Złamanie przestrzennej symetrii inwersji

$$H_K = -\frac{\sqrt{3}}{2} at\left(\sigma_x k_x + \sigma_y k_y\right) + \frac{1}{2}\Delta\sigma_z = -\frac{\sqrt{3}}{2} at\vec{\sigma}\cdot\vec{k} + \frac{1}{2}\Delta\sigma_z$$

Grafen - struktura pasmowa

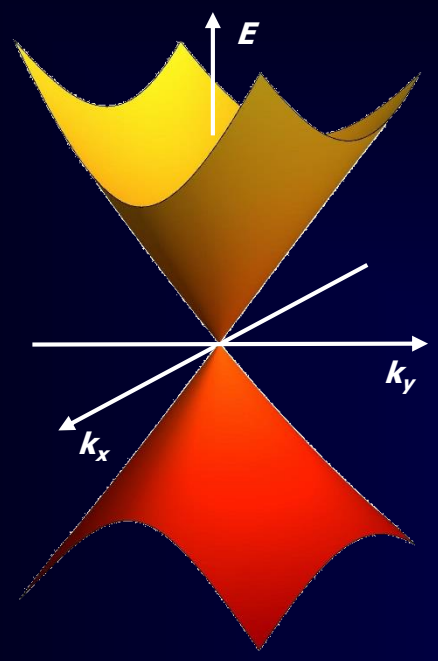
$$H_K = -\frac{\sqrt{3}}{2} at\vec{\sigma}\cdot\vec{k} + \frac{1}{2}\Delta\sigma_z$$

$$H_K = \begin{pmatrix} \frac{1}{2}\Delta & -\frac{\sqrt{3}}{2}at\left(k_x - ik_y\right) \\ -\frac{\sqrt{3}}{2}at\left(k_x + ik_y\right) & -\frac{1}{2}\Delta \end{pmatrix}$$

$$E = \pm\sqrt{\frac{3}{4}(at)^2\left(k_x^2 + k_y^2\right) + \frac{1}{2}\Delta}$$

Grafen - struktura pasmowa

półmetal

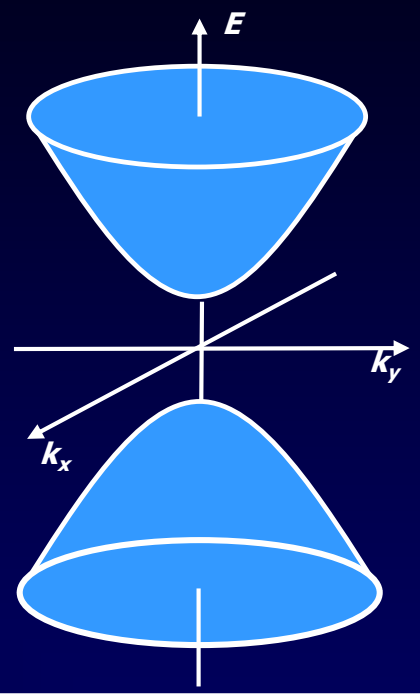


Bezmasowe Fermiony Diraca

$$E = \pm \frac{\sqrt{3}}{2} at \sqrt{\left(k_x^2 + k_y^2\right)}$$

$$E = \pm v_F \hbar \ |k|$$

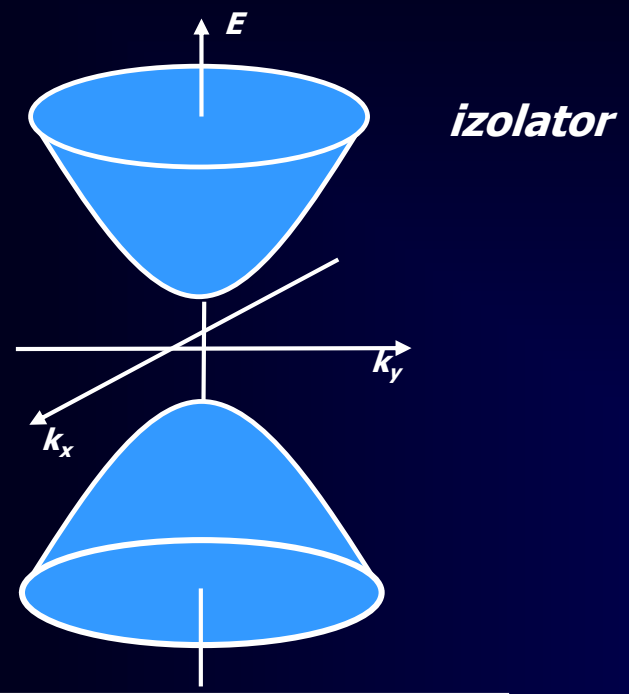
izolator



Fermiony Diraca z masą

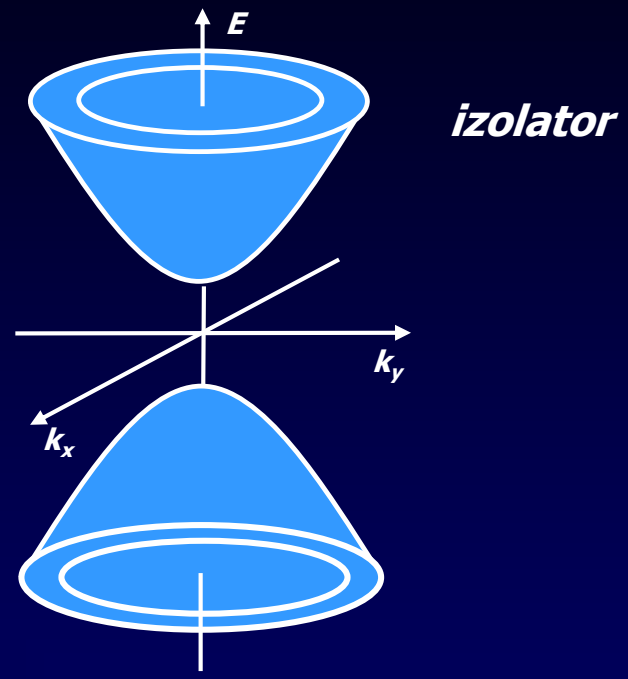
$$E = \pm \sqrt{\frac{3}{4}(at)^2\left(k_x^2 + k_y^2\right) + \frac{1}{2}\Delta}$$

Grafen/usunięcie degeneracji



Fermiony Diraca z masą

$$E = \pm\sqrt{\frac{3}{4}(at)^2\left(k_x^2 + k_y^2\right) + \frac{1}{2}\Delta}$$

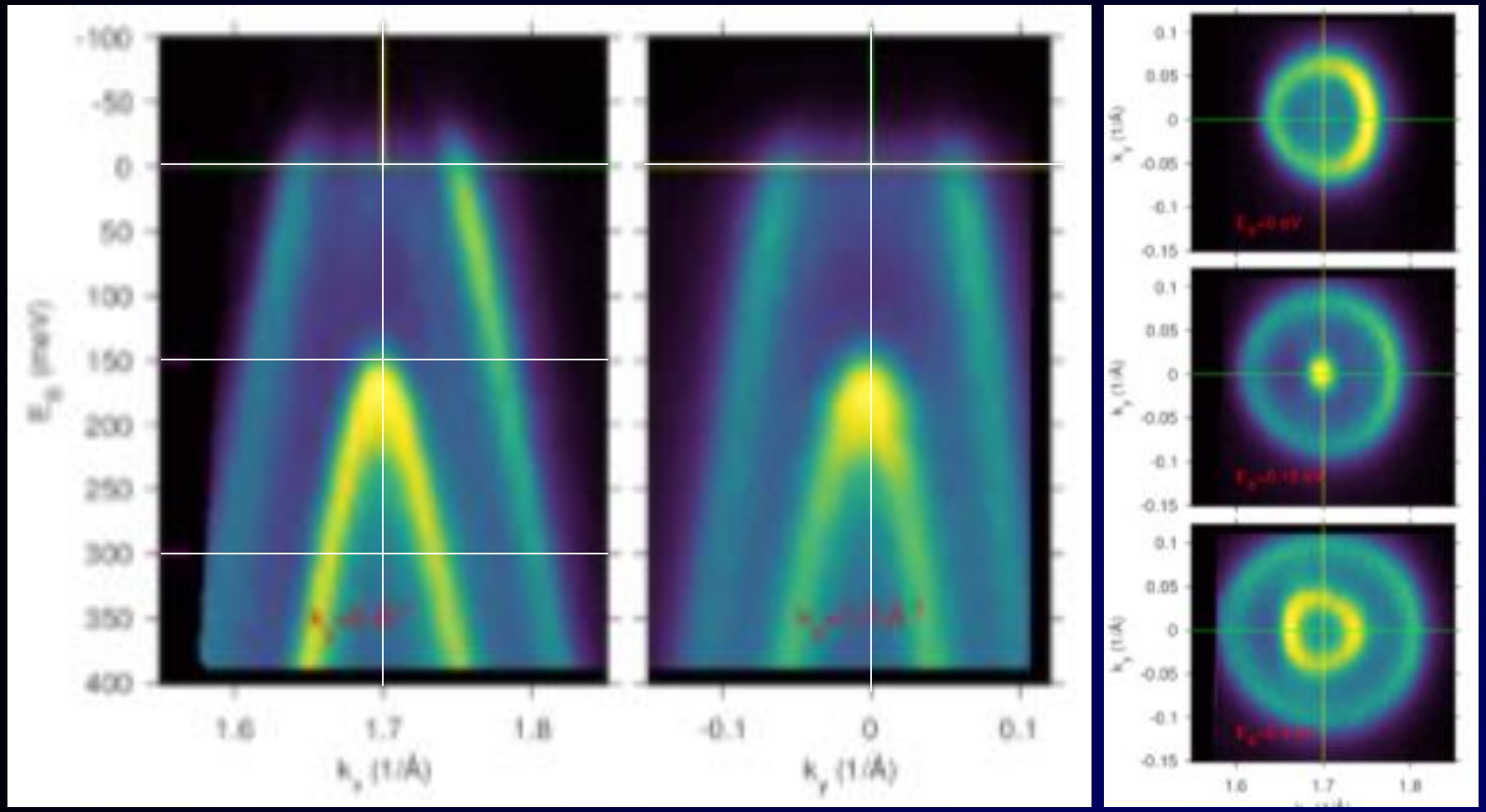


Fermiony Diraca z masą

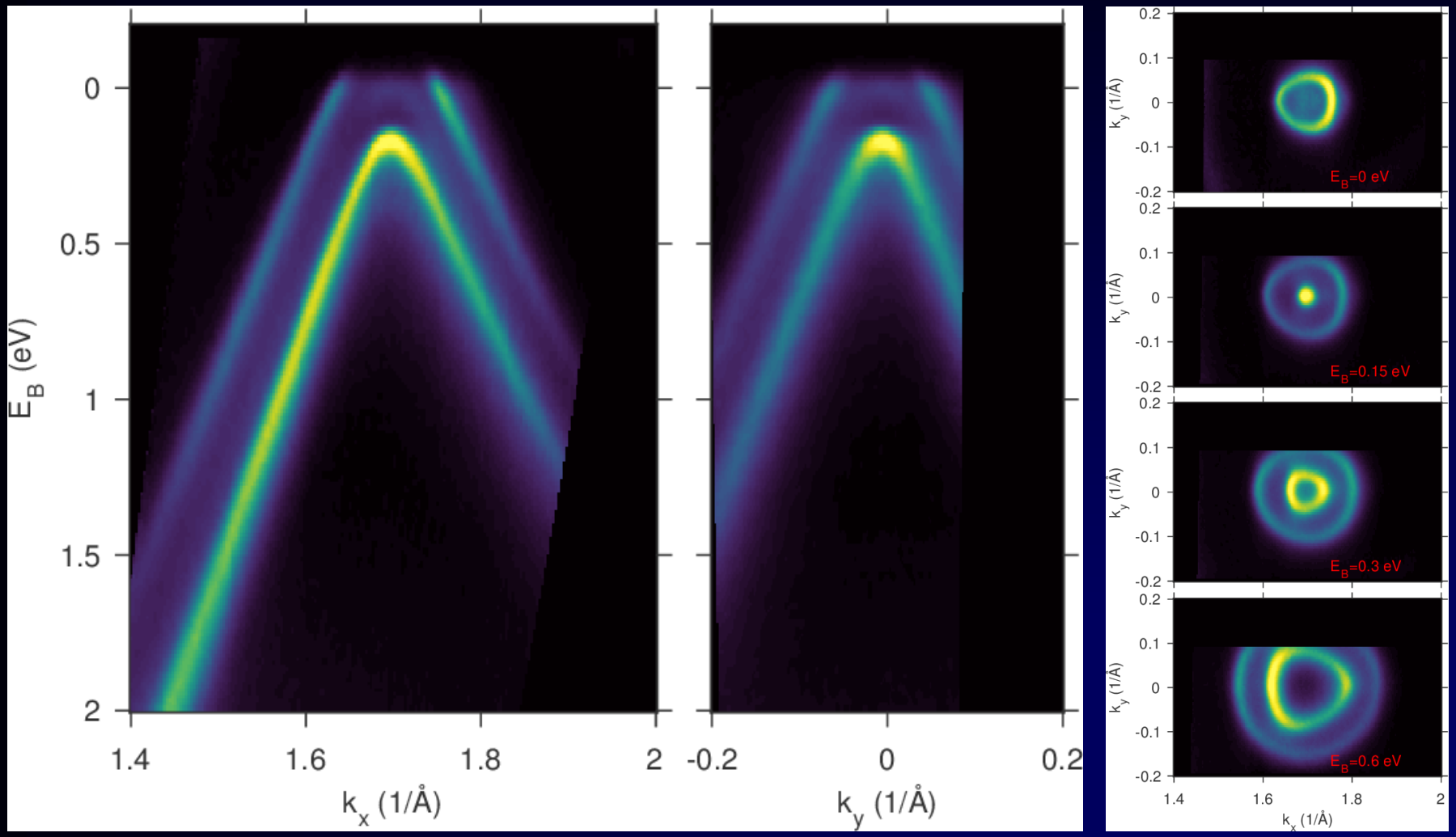
$$E = \pm\sqrt{\frac{3}{4}(at)^2\left(k_x^2 + k_y^2\right) + \frac{1}{2}\Delta}$$

Struktura pasmowa jest symetryczna wokół punktu k=0
Pasma są podwójnie zdegenerowane

ARPES – kątowo rozdzielcza fotoemisja elektronów



ARPES – kątowo rozdzielcza fotoemisja elektronów



UNIWERSYTET ŁÓDZKI

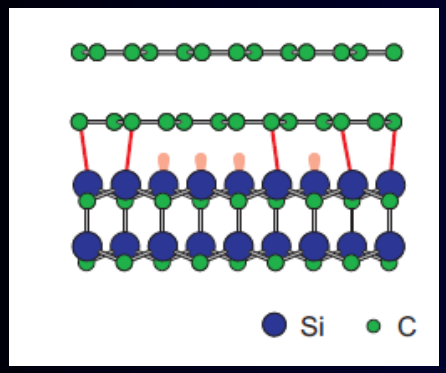
grafen/4H-SiC(0001) – $H_{interkalacja}$

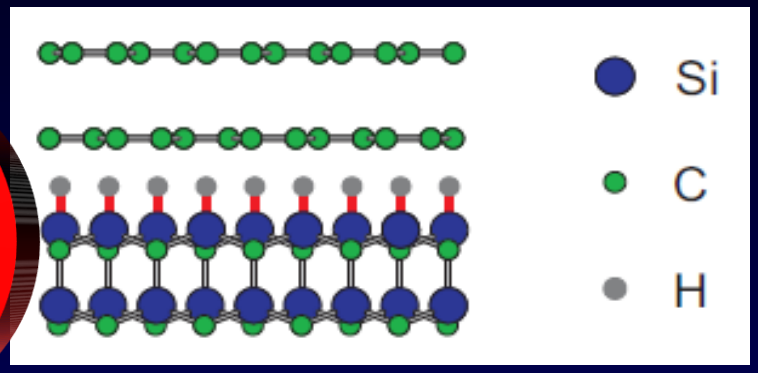
grafen/SiC

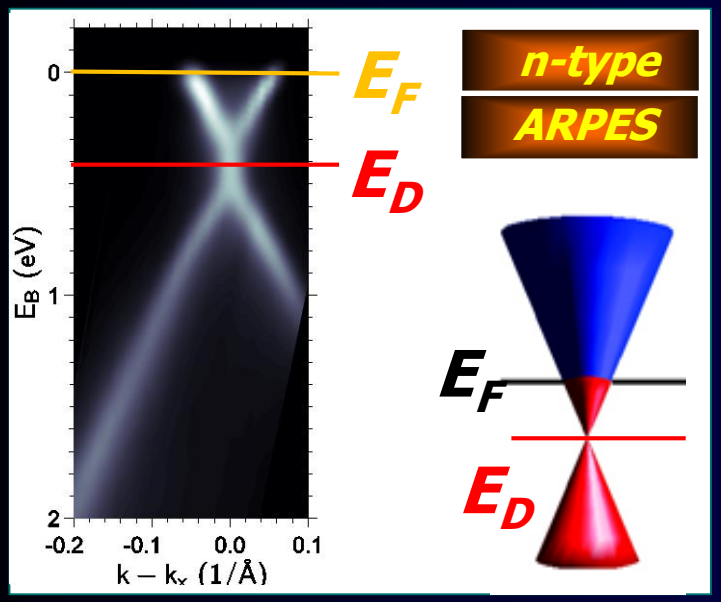
J.M. Baranowski, M. Możdżonek, P. Dąbrowski, K. Grodecki, P. Osewski, W. Kozłowski, M. Kopciuszyński, M. Jałochowski, W. Strupiński

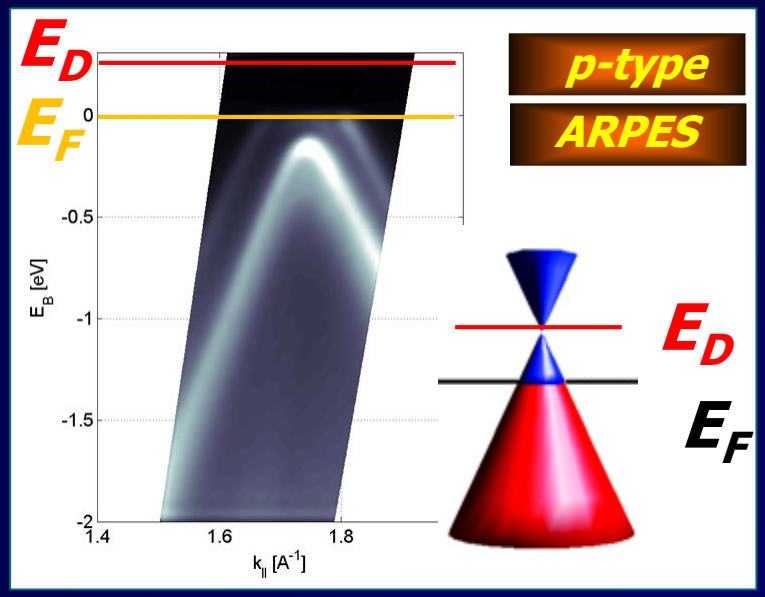
Graphene, 2, 115120 (2013)

grafen/SiC+H





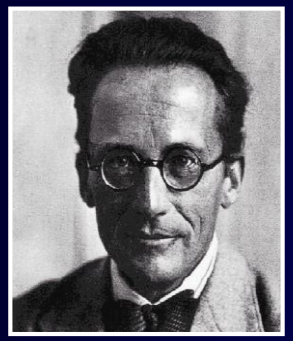




Grafen - struktura pasmowa

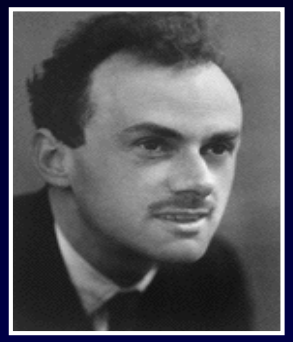
$$-\frac{\hbar^2}{2m_e^*}\Delta\psi + \hat{V}\psi = E\psi$$

$$-\frac{\hbar^2}{2m_h^*}\Delta\psi + \hat{V}\psi = E\psi$$



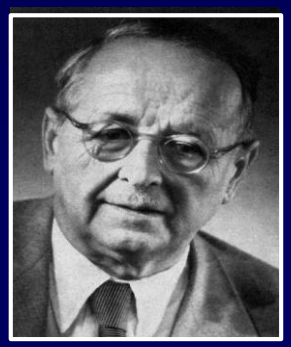
E. Schrodinger

fermiony z masą



P. A. M. Dirac

bezmasowe fermiony



H. Weyl

Równanie Diraca

$$H_K = c\vec{\alpha}\cdot\vec{P} + \beta m_0 c^2$$

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \qquad \vec{\beta} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \qquad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\left[-i\hbar c\vec{\alpha}\vec{\nabla} + \beta mc^2\right]\begin{pmatrix} \Psi_1(\vec{r},t) \\ \Psi_2(\vec{r},t) \\ \Psi_3(\vec{r},t) \\ \Psi_4(\vec{r},t) \end{pmatrix} = i\hbar\frac{\partial}{\partial t}\begin{pmatrix} \Psi_1(\vec{r},t) \\ \Psi_2(\vec{r},t) \\ \Psi_3(\vec{r},t) \\ \Psi_4(\vec{r},t) \end{pmatrix}$$

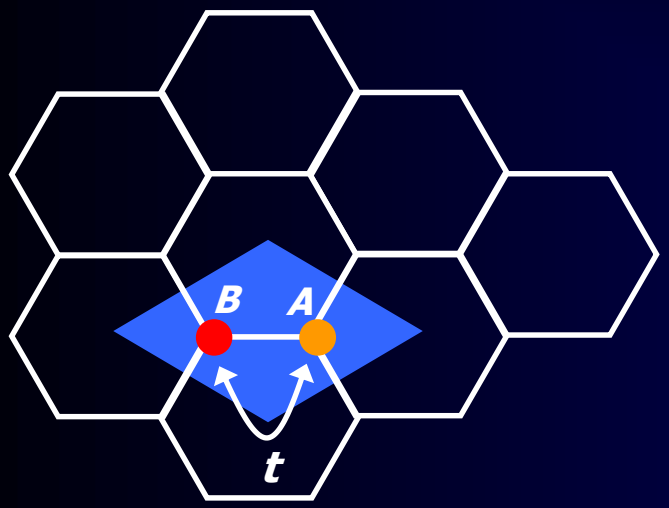
Grafen (Dirac -> Weyl)

$$H_K = c\vec{\alpha}\cdot\vec{P} + \beta m_0 c^2 \;\Rightarrow\; c\vec{\alpha}\cdot\vec{P}$$

$$\hbar v_F \begin{pmatrix} 0 & k_x - ik_y \\ k_x + ik_y & 0 \end{pmatrix} \begin{pmatrix} \varphi_A(r) \\ \varphi_B(r) \end{pmatrix} = E \begin{pmatrix} \varphi_A(r) \\ \varphi_B(r) \end{pmatrix}$$

$$\hbar v_F (\vec{\sigma}\cdot\vec{k})\Psi(r) = E\Psi(r)$$

Co to jest pseudospin



Dwie podsieci A i B – dwa stopnie swobody

elektron scharakteryzowany jest przez amplitudę prawdopodobieństwa, że znajduje się w danej podsieci

Baza dwóch podsieci przypomina układ ze spinem

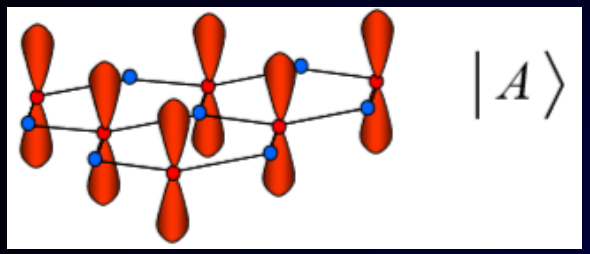
$$|A\rangle \quad |B\rangle$$

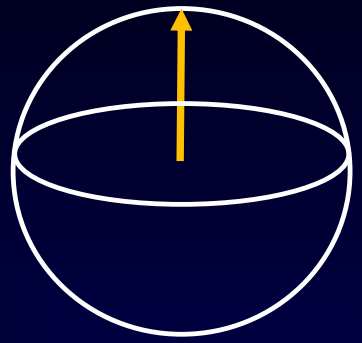
$$|A\rangle + |B\rangle e^{i\theta}$$

$$|\uparrow\rangle \quad |\downarrow\rangle$$

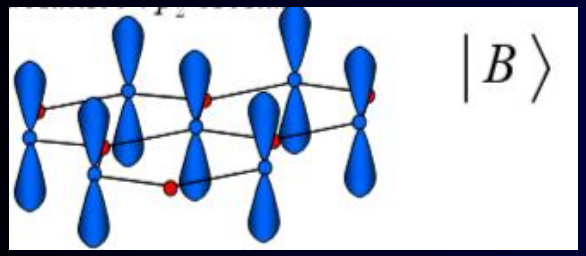
$$\cos\frac{\vartheta}{2}|\uparrow\rangle + \sin\frac{\vartheta}{2} e^{i\varphi}|\downarrow\rangle$$

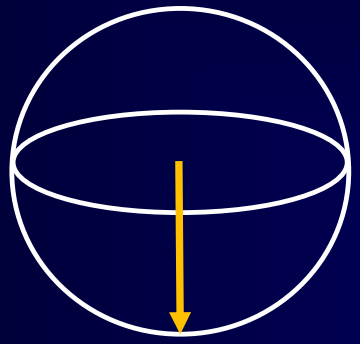
Co to jest pseudospin





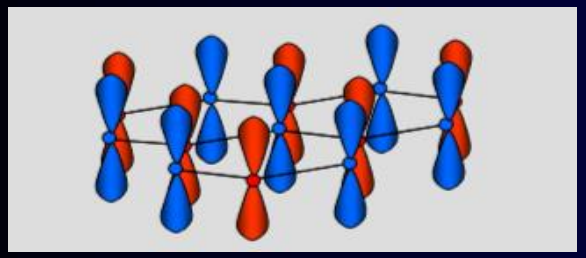
$$|A\rangle$$

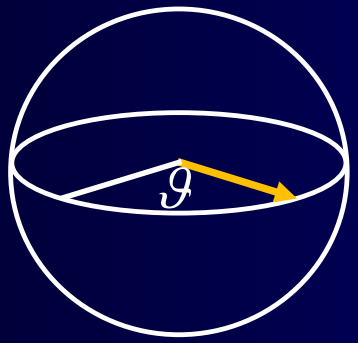




$$|B\rangle$$

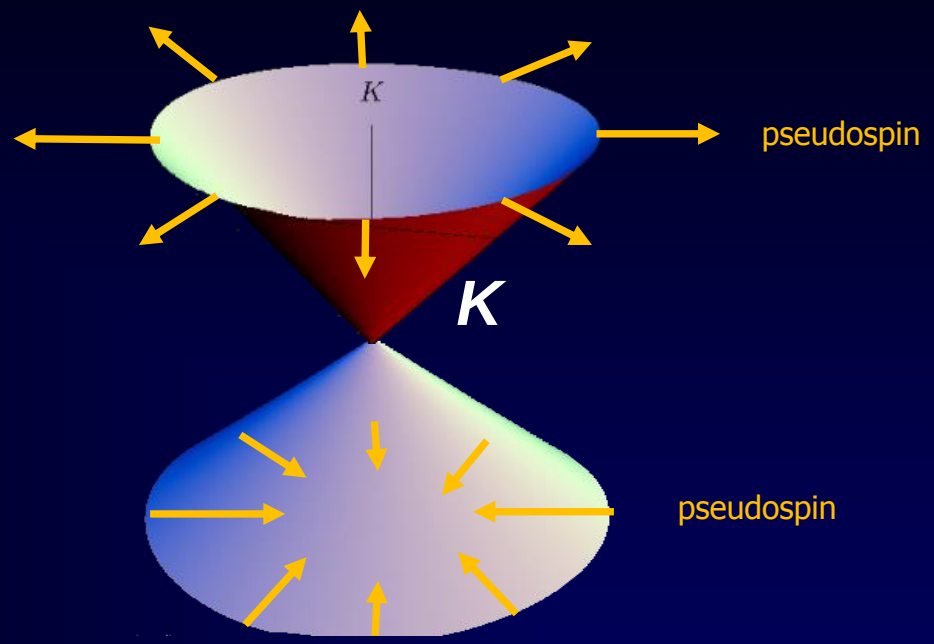
$$\begin{pmatrix} \varphi_A(r) \\ \varphi_B(r) \end{pmatrix}$$





$$|A\rangle + e^{i\theta}|B\rangle$$

Co to jest pseudospin



przeskok z B do A

$$\hbar v_F \begin{pmatrix} 0 & k_x - ik_y \\ k_x + ik_y & 0 \end{pmatrix} \begin{pmatrix} \varphi_A(r) \\ \varphi_B(r) \end{pmatrix} = E \begin{pmatrix} \varphi_A(r) \\ \varphi_B(r) \end{pmatrix}$$

przeskok z A do B

Symetrie

symetria translacji

$$\hat{\mathrm{T}}_{\vec{R}} f(\vec{r}) = f(\vec{r} + \vec{R})$$

$$\left[\hat{H}, \hat{\mathrm{T}}_R\right] = 0$$

symetria inwersji przestrzennej

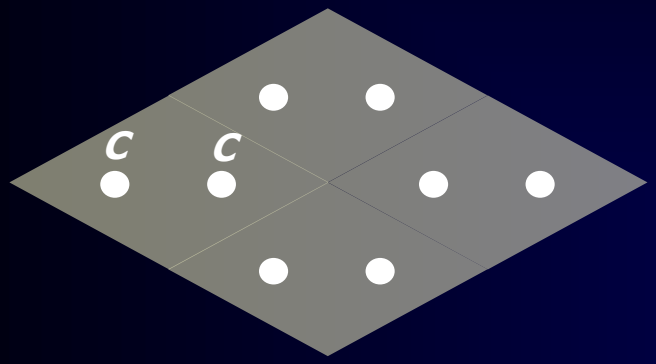
$$\hat{\mathrm{I}}\vec{r} = -\vec{r}$$

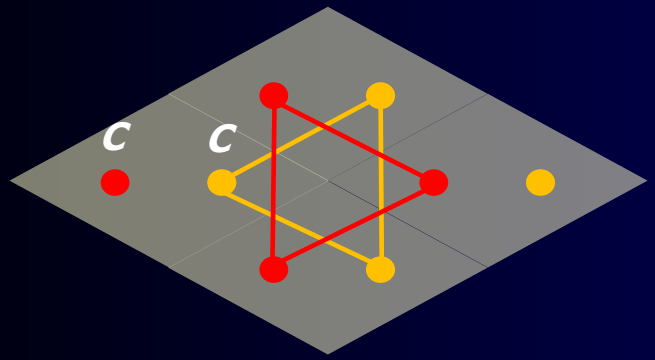
$$E(k) = E(-k)$$

Energia cząstki poruszającej się w prawo jest równa energii cząstki poruszającej się w lewo. Zasada słuszna dla cząstek ze spinem w górę i w dół.

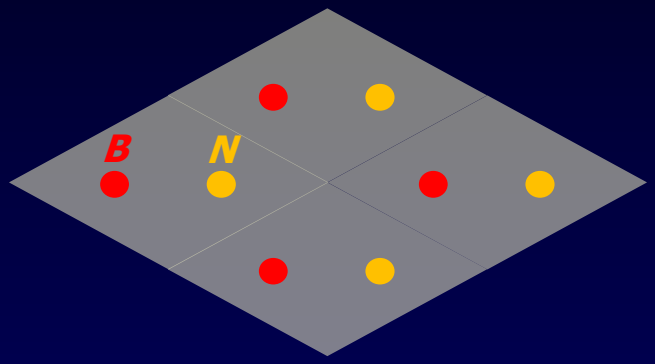
Symetria inwersji przestrzennej

grafen

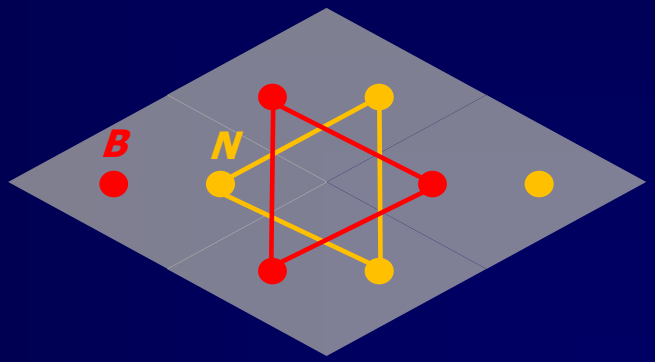




h-BN



Symetria inwersji przestrzennej złamana



Symetrie

symetria translacji

$$\hat{\mathrm{T}}_{\vec{R}} f(\vec{r}) = f(\vec{r} + \vec{R})$$

$$\left[\hat{H}, \hat{\mathrm{T}}_R\right] = 0$$

symetria inwersji przestrzennej

$$\hat{\mathrm{I}}\vec{r} = -\vec{r}$$

$$E(k) = E(-k)$$

Energia cząstki poruszającej się w prawo jest równa energii cząstki poruszającej się w lewo. Zasada słuszna dla cząstek ze spinem w górę i w dół.

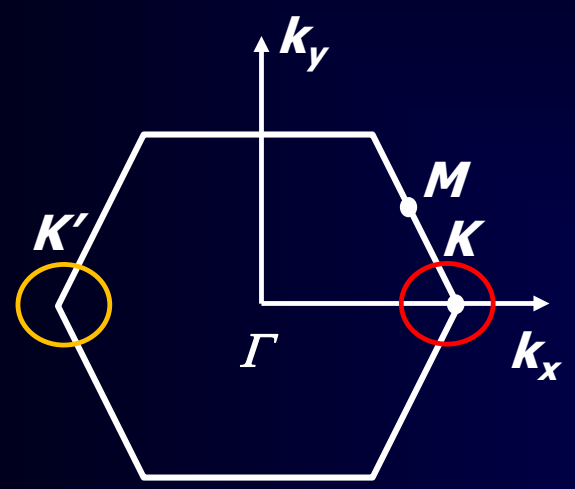
symetria inwersji czasu

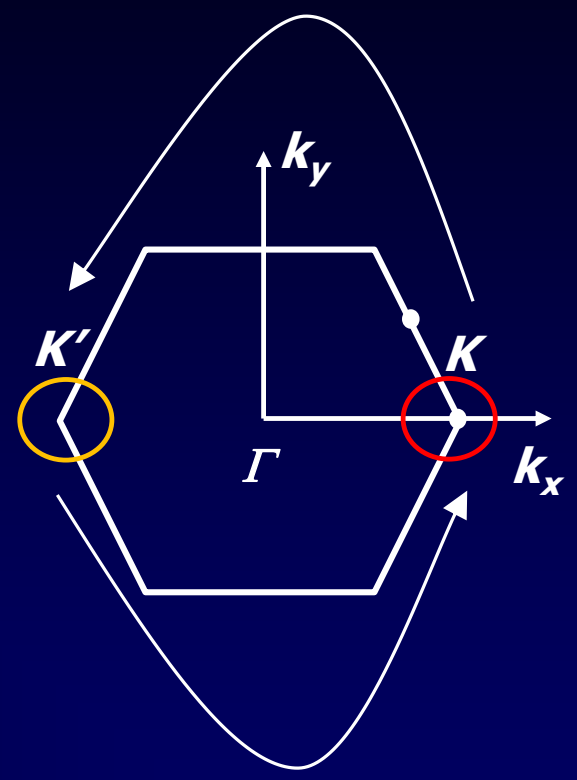
$$t \rightarrow -t$$

$$E(k,\uparrow) = E(-k,\downarrow)$$

Energia cząstki poruszającej się w prawo ze spinem do góry jest równa energii cząstki poruszającej się w lewo ze spinem w dół

Symetria inwersji czasu





$$t \rightarrow -t$$

$$E(k,\uparrow) = E(-k,\downarrow)$$

Symetria inwersji czasu

$$H_K = v_F \hbar \vec{\sigma} \cdot \vec{k}$$

$$\vec{\sigma} = \left(\sigma_x, \sigma_y\right)$$

$$H_K = v_F \hbar \begin{pmatrix} 0 & k_x - ik_y \\ k_x + ik_y & 0 \end{pmatrix}$$

$$H_{K'} = v_F \hbar \vec{\sigma}' \cdot \vec{k}$$

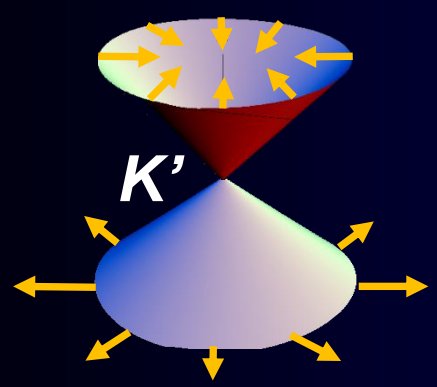
$$\vec{\sigma}' = \left(-\sigma_x, \sigma_y\right)$$

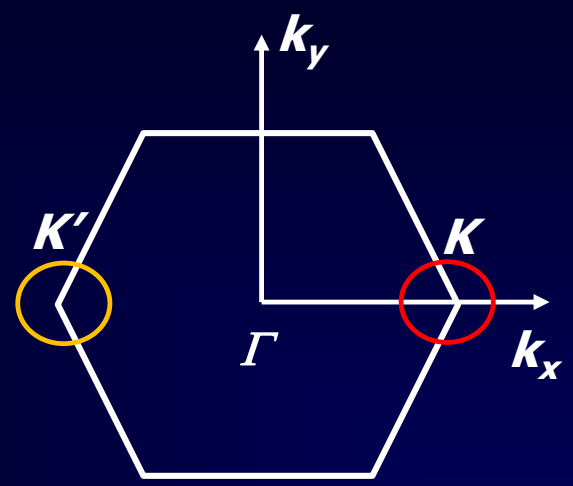
$$H_{K'} = v_F \hbar \begin{pmatrix} 0 & -k_x - ik_y \\ -k_x + ik_y & 0 \end{pmatrix}$$

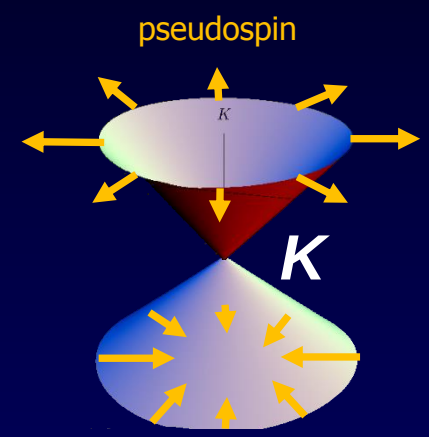
Pełny Hamiltonian

$$v_F\hbar\begin{pmatrix} 0 & k_x - ik_y & 0 & 0 \\ k_x + ik_y & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_x - ik_y \\ 0 & 0 & -k_x + ik_y & 0 \end{pmatrix}\begin{pmatrix} \varphi_{AK}(r) \\ \varphi_{BK}(r) \\ \varphi_{AK'}(r) \\ \varphi_{BK'}(r) \end{pmatrix} = E\begin{pmatrix} \varphi_{AK}(r) \\ \varphi_{BK}(r) \\ \varphi_{AK'}(r) \\ \varphi_{BK'}(r) \end{pmatrix}$$

Struktura elektronowa grafenu



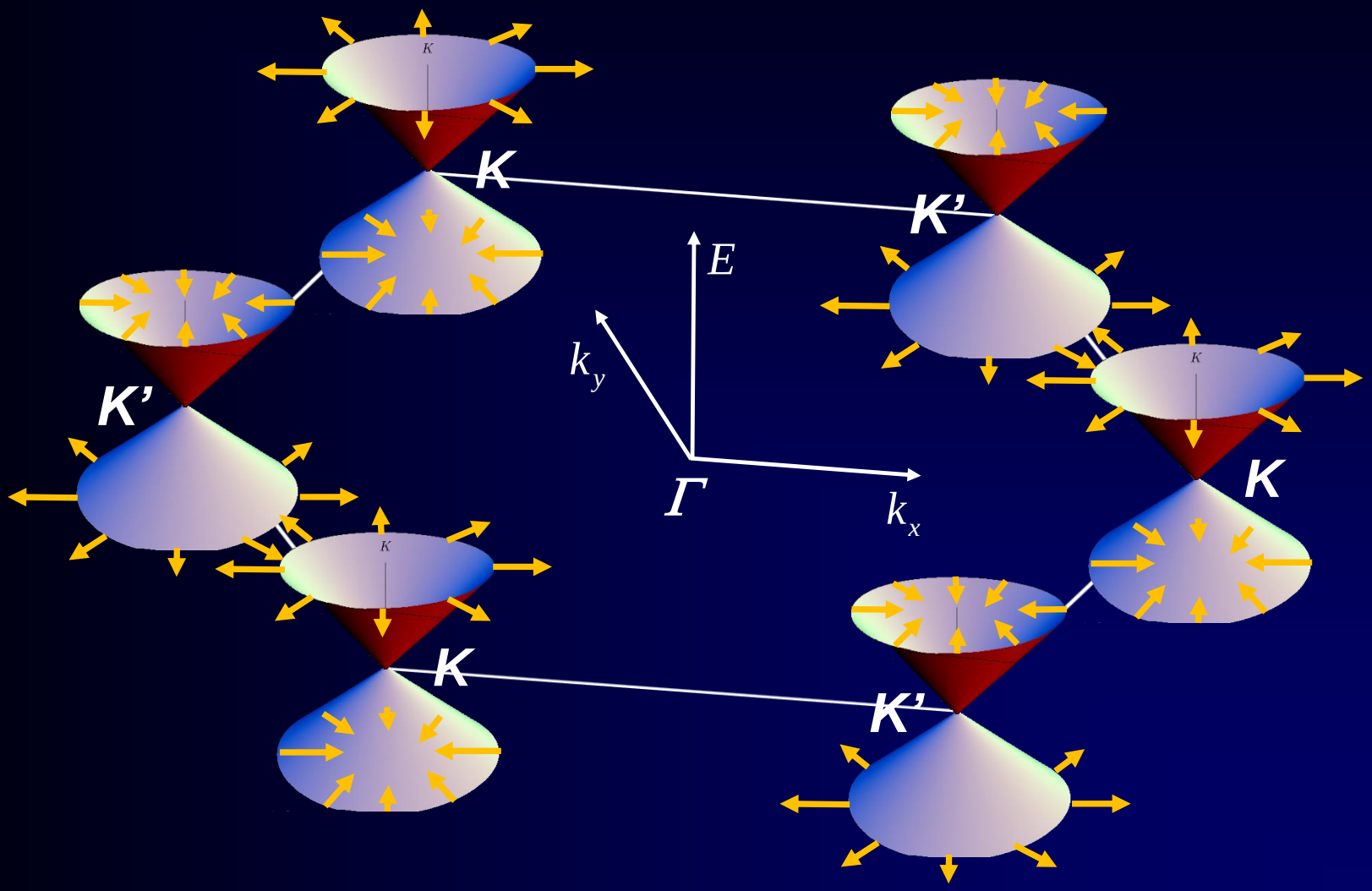




$$v_F\hbar\begin{pmatrix} 0 & -k_x - ik_y \\ -k_x + ik_y & 0 \end{pmatrix}\begin{pmatrix} \varphi_{AK'}(r) \\ \varphi_{BK'}(r) \end{pmatrix} = E\begin{pmatrix} \varphi_{AK'}(r) \\ \varphi_{BK'}(r) \end{pmatrix}$$

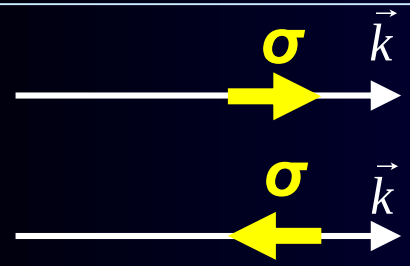
$$v_F\hbar\begin{pmatrix} 0 & k_x - ik_y \\ k_x + ik_y & 0 \end{pmatrix}\begin{pmatrix} \varphi_{AK}(r) \\ \varphi_{BK}(r) \end{pmatrix} = E\begin{pmatrix} \varphi_{AK}(r) \\ \varphi_{BK}(r) \end{pmatrix}$$

Struktura elektronowa grafenu



Grafen – skrętność

$$H_K = v_F \hbar \vec{\sigma} \cdot \vec{k} = v_F \hbar k \vec{\sigma} \vec{n}$$



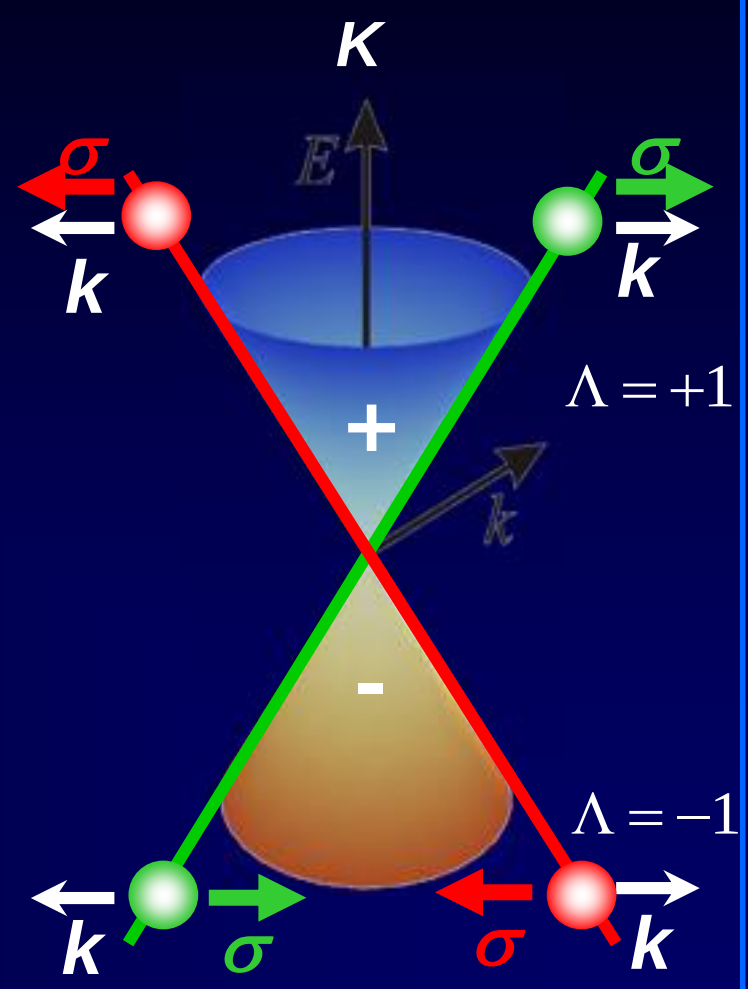
Skrętność

$$\Lambda = \frac{\vec{\sigma}\vec{k}}{k} = +1$$ ***prawoskrętne***

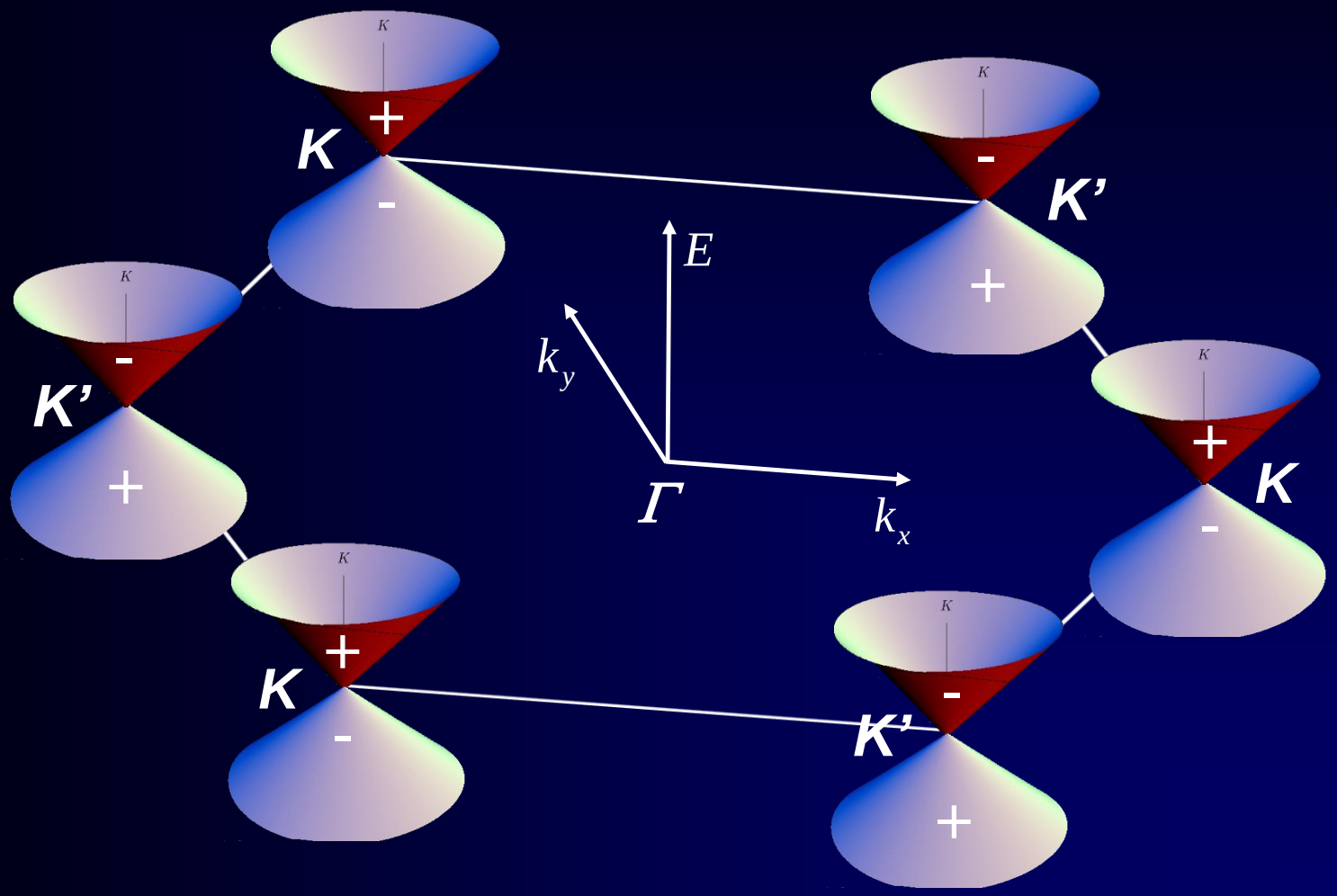
$$\Lambda = \frac{\vec{\sigma}\vec{k}}{k} = -1$$ ***lewoskrętne***

Elektrony i dziury z tym samym wektorem falowym mają przeciwne prędkości grupowe

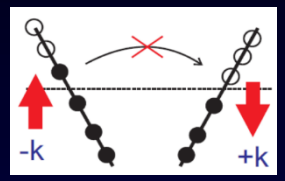
$$\vec{v} = d\varepsilon / d(\hbar\vec{k}) = \begin{cases} +v_F\vec{n} & e \\ -v_F\vec{n} & h \end{cases}$$

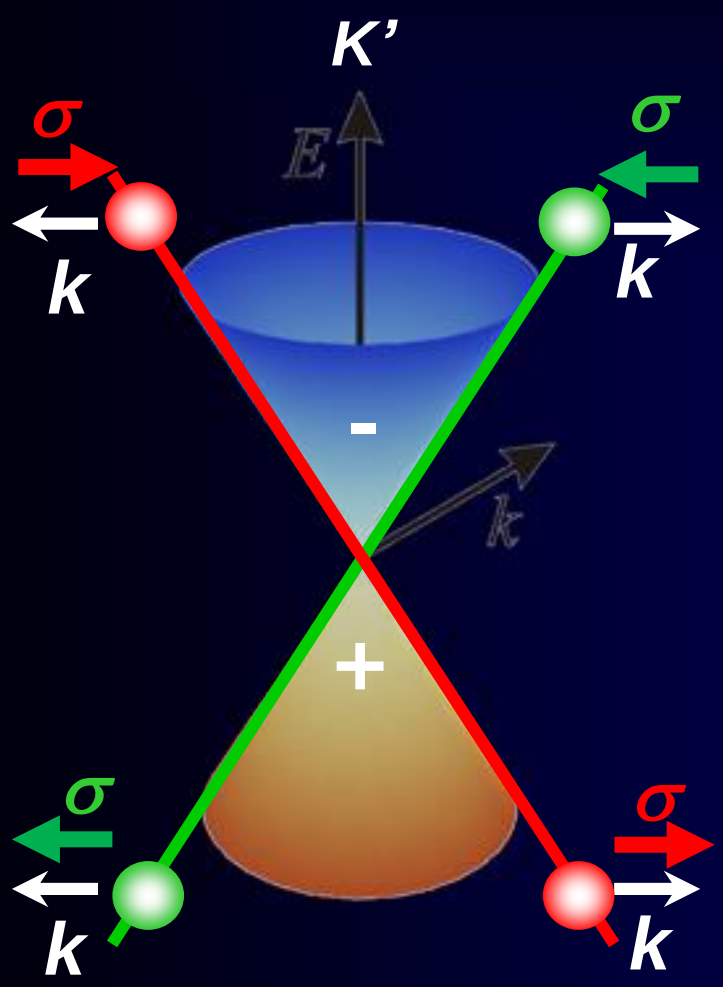


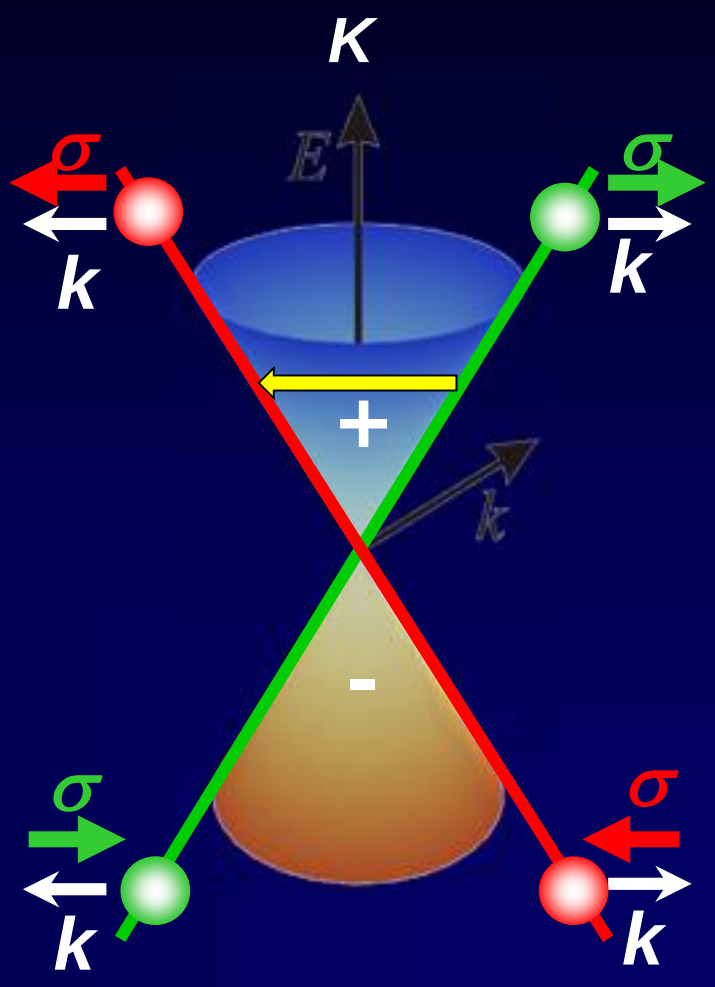
Grafen – skrętność



Grafen – rozpraszanie



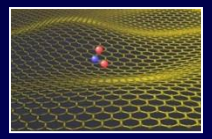
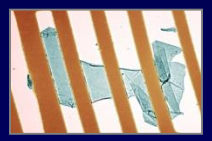




Grafen - struktura pasmowa

Ruchliwość nośników
Przewodnictwo elektryczne

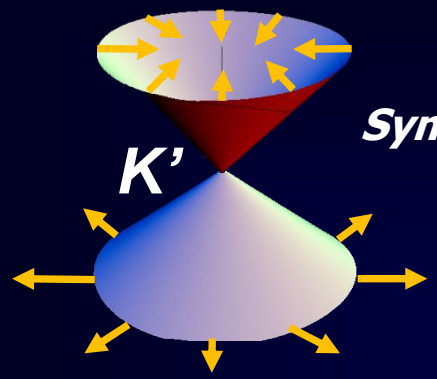
$$\mu = \frac{v}{E} \qquad E = \frac{U}{d}\left[\frac{V}{cm}\right] \qquad v = \frac{d}{t}\left[\frac{cm}{s}\right] \Rightarrow \sigma \approx en\mu$$

$10\,000 - 15\,000\, cm^2V^{-1}s^{-1}$	**Grafen na SiO_2**	
$200\,000\, cm^2V^{-1}s^{-1}$	**Grafen na SiO_2 bez zanieczyszczeń**	
$1\,000\,000\, cm^2V^{-1}s^{-1}$	**Grafen zawieszony**	

T=300 K
Bez domieszkowania

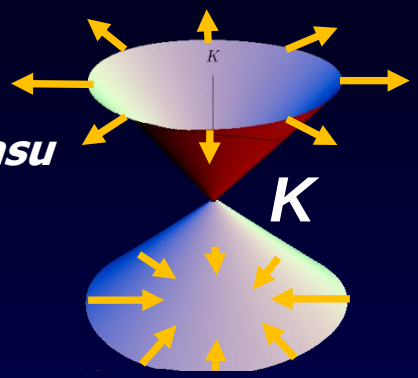
Si	$1\,300$	$cm^2V^{-1}s^{-1}$	$GaAs$	$8\,800$	$cm^2V^{-1}s^{-1}$
Ge	$3\,800$	$cm^2V^{-1}s^{-1}$	$InSb$	$78\,000$	$cm^2V^{-1}s^{-1}$

Valleytronika



Symetria inwersji czasu

$$K \rightarrow K'$$



Potrzebujemy wielkości fizycznej która odróżni K od K'

Spin – jako przykład

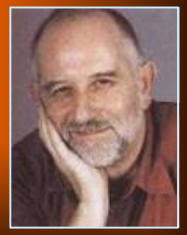
Spin do góry i spin do dołu przechodzą w siebie na skutek inwersji czasu. Kierunek spinu rozróżniamy za pomocą przeciwnych momentów magnetycznych. Moment magnetyczny jest pseudowektorem, który jest nieparzysty na skutek symetrii inwersji czasu.

Medium do detekcji – pole magnetyczne.

Pseudospin

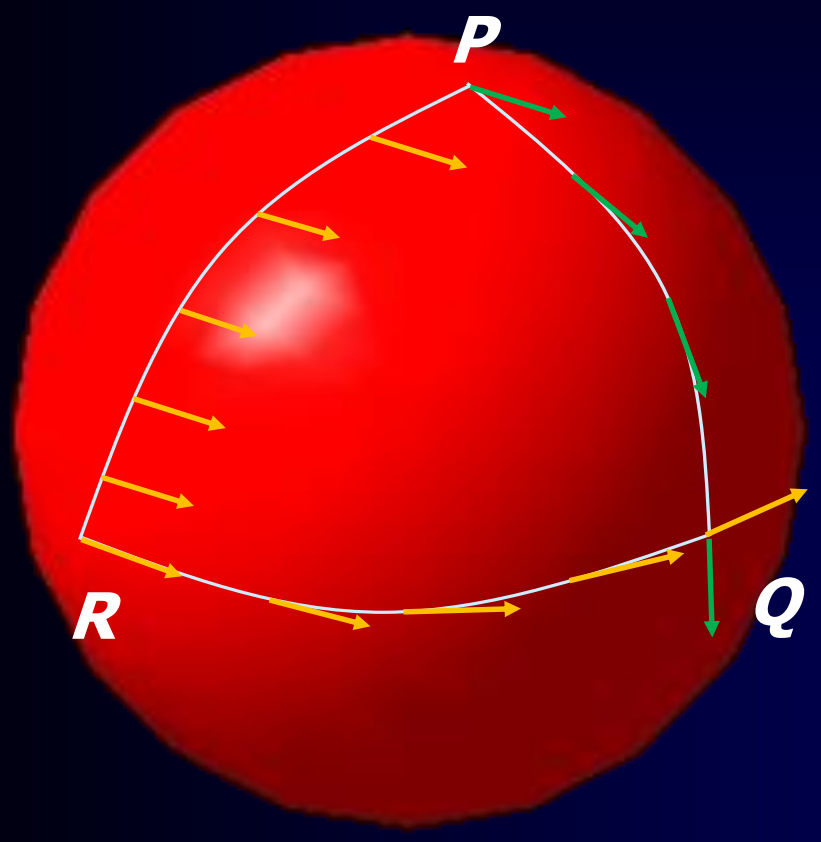
Wielkości fizyczne które są nieparzyste na skutek symetrii inwersji czasu nadają się do detekcji pseudospinu.

Ω krzywizna Berry'ego



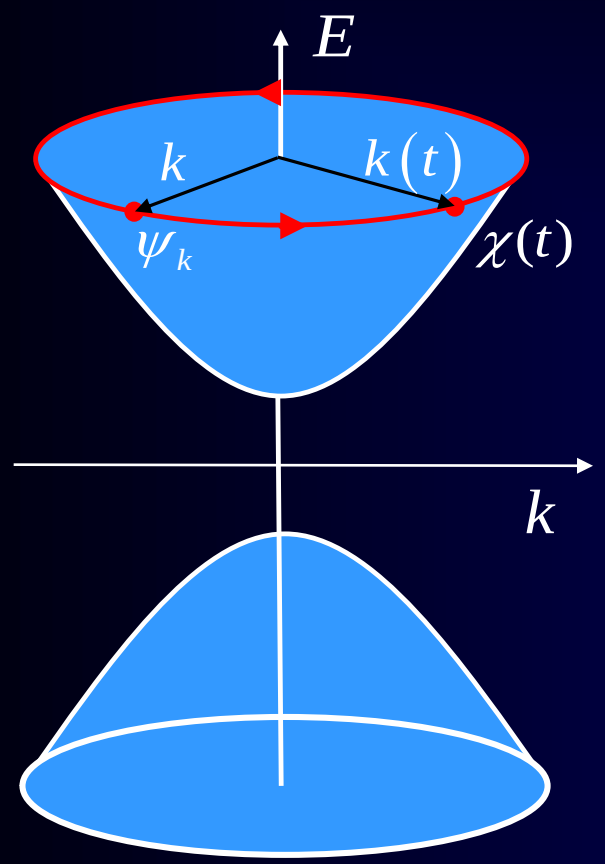
P. Berry

Krzywizna Berry'ego



Koneksja afinczna wyraża ideę prześnienia równoległego wektorów stycznych wzdłuż krzywych. Miarą krzywizny jest stopień rozbieżności przy przemieszczaniu wektorów wzdłuż krzywych.

Faza Berry'ego



$$|\psi_k\rangle \Rightarrow |\chi(t)\rangle$$

Ewolucja stanu na skutek adiabatycznej rotacji wektora falowego. Rotację modelujemy za pomocą k(t)

Przyjmujemy, że w dowolnej chwili czasu t stan $|\chi(t)\rangle$

pozostaje stanem własnym hmiltonianu $\hat{h}(k(t))$

który zależy od czasu tylko poprzez k(t).

Faza Berry'ego

$$i\hbar \frac{\partial |\chi(t)\rangle}{\partial t} = E_{\vec{k}(t)} |\chi(t)\rangle$$

$$|\chi(t)\rangle = e^{-i\theta(t)} \left|\psi_{\vec{k}(t)}(t)\right\rangle$$

Rozwiązanie musi być proporcjonalne do

$$\left|\psi_{\vec{k}(t)}(t)\right\rangle$$

$$\left\langle \psi_{\vec{k}(t)}(t) \middle| \psi_{\vec{k}(t)}(t) \right\rangle = 1$$

$\theta(t)$ *jest nieznaną fazą*

Faza Berry'ego

$$i\hbar \frac{\partial \left| \psi_{\vec{k}(t)} \right\rangle}{\partial t} + \hbar \dot{\theta}(t) \left| \psi_{\vec{k}(t)} \right\rangle = E_{\vec{k}(t)} \left| \psi_{\vec{k}(t)} \right\rangle$$

$$\dot{\theta}(t) = \frac{1}{\hbar} E_{\vec{k}(t)} - i \left\langle \psi_{\vec{k}(t)} \middle| \frac{\partial}{\partial t} \middle| \psi_{\vec{k}(t)} \right\rangle$$

$$\theta(t) = \frac{1}{\hbar} \int_0^t E_{\vec{k}(t')} dt' - i \int_0^t \left\langle \psi_{\vec{k}(t')} \middle| \frac{\partial}{\partial t'} \middle| \psi_{\vec{k}(t')} \right\rangle dt'$$

Faza na skutek ewolucji adiabatycznej

Koneksja Berry'ego

$$\gamma = -i\int_0^{t_{cykl}} \left\langle \psi_{\vec{k}(t')} \middle| \frac{\partial}{\partial t'} \middle| \psi_{\vec{k}(t')} \right\rangle dt' = -i\oint \left\langle \psi_{\vec{k}(t')} \middle| \vec{\nabla}_{\vec{k}} \middle| \psi_{\vec{k}(t')} \right\rangle \cdot d\vec{k}$$

Definiujemy potencjał wektorowy Koneksja Berry'ego

$$\vec{A}_{\vec{k}} - i\left\langle \psi_{\vec{k}(t')} \middle| \vec{\nabla}_{\vec{k}} \middle| \psi_{\vec{k}(t')} \right\rangle$$

Krzywizna Berry'ego

Fazę Berry'ego można przedstawić w formie analogicznej jak w przypadku strumienia magnetycznego

$$\gamma = \oint \vec{A}_{\vec{k}} \cdot d\vec{k} = \int \left(\vec{\nabla}_{\vec{k}} \times \vec{A}_{\vec{k}} \right)_z dk_x dk_y$$

Analogies

Berry curvature $\Omega(\vec{\lambda})$	Magnetic field $B(\vec{r})$
Berry connection $\langle \psi \| i \frac{\partial}{\partial \lambda} \| \psi \rangle$	Vector potential $A(\vec{r})$
Geometric phase $\int d\lambda \langle \psi \| i \frac{\partial}{\partial \lambda} \| \psi \rangle = \iint d^2\lambda \ \Omega(\vec{\lambda})$	Aharonov-Bohm phase $\oint dr \ A(\vec{r}) = \iint d^2r \ B(\vec{r})$
Chern number $\iint d^2\lambda \ \Omega(\vec{\lambda}) = \text{integer}$	Dirac monopole $\iint d^2r \ B(\vec{r}) = \text{integer} \ h/e$

$$\left(\vec{\nabla}_{\vec{k}} \times \vec{A}_{\vec{k}} \right)_z$$

Pełni rolę pola magnetycznego prostopadłego do powierzchni zamkniętej przez krzywą całkowania

Krzywizna Berry'ego

$$\Omega = \vec{\nabla}_{\vec{k}} \times \vec{A}_{\vec{k}}$$

Valleytronika

$$\vec{v}_n(\vec{k}) = \frac{1}{\hbar}\frac{\partial E_n(\vec{k})}{\partial \vec{k}}$$

Prędkość związana energią pasma

$$\vec{v}_n(\vec{k}) = \frac{1}{\hbar}\frac{\partial E_n(\vec{k})}{\partial \vec{k}} - \frac{e}{\hbar}\vec{E}\times\vec{\Omega}_n(\vec{k})$$

Pełny opis prędkości

Kiedy krzywizna Berry'ego nie może być pominięta ?

Valleytronika – 2D sieć heksagonalna

- *symetria odwrócenia czasu* $\vec{\Omega}_n(\vec{k}) = -\vec{\Omega}_n(-\vec{k})$
- *przestrzenna inwersja symetrii* $\vec{\Omega}_n(\vec{k}) = \vec{\Omega}_n(-\vec{k})$

W krysztale z jednoczesną symetrią odwrócenia czasu i symetrią inwersji przestrzennej krzywizna Berryego znika

$$\vec{v}_n(\vec{k}) = \frac{1}{\hbar}\frac{\partial E_n(\vec{k})}{\partial \vec{k}}$$

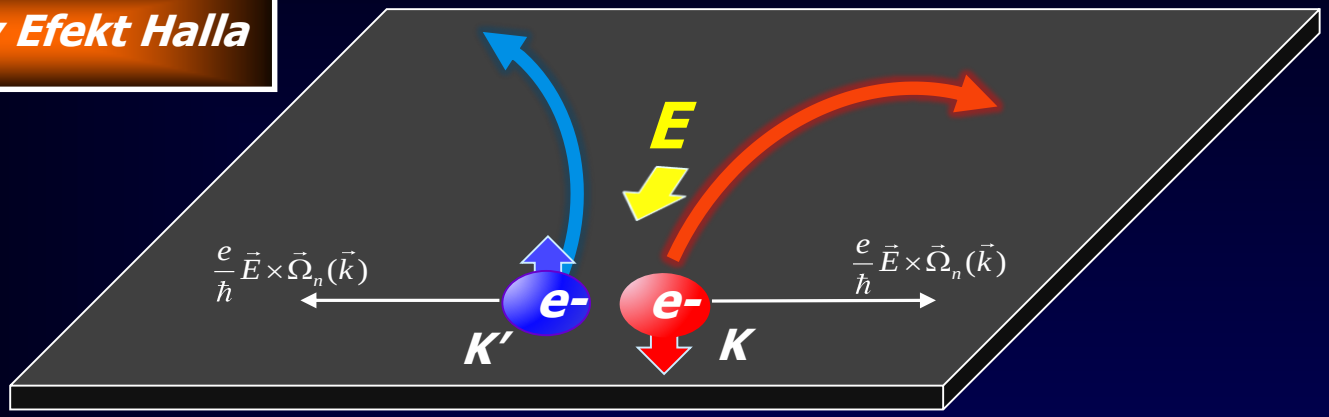
W krysztale w którym symetria odwrócenia czasu lub symetrią inwersji przestrzennej jest złamana krzywizna Berryego pozostaje

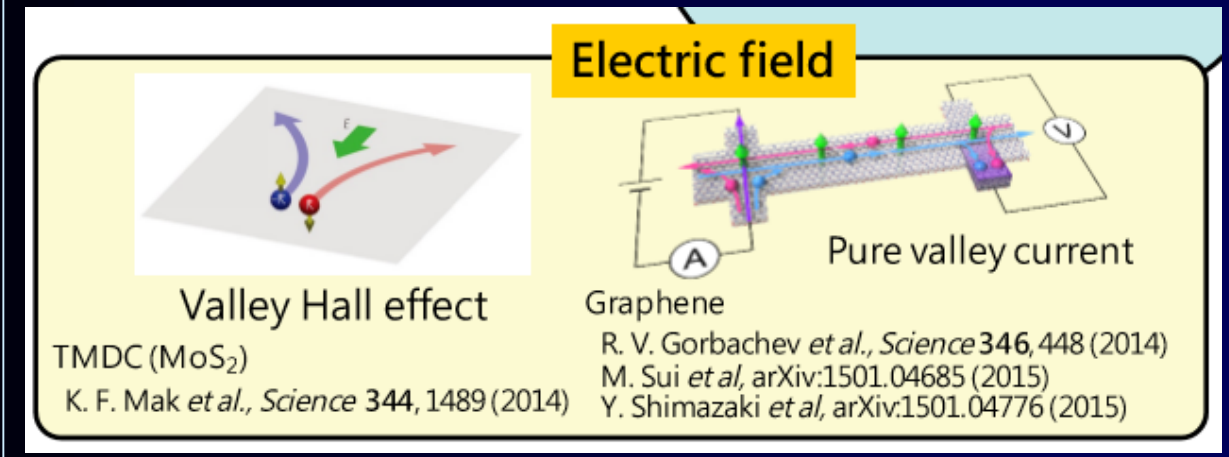
$$\vec{v}_n(\vec{k}) = \frac{1}{\hbar}\frac{\partial E_n(\vec{k})}{\partial \vec{k}} - \frac{e}{\hbar}\vec{E}\times\vec{\Omega}_n(\vec{k})$$

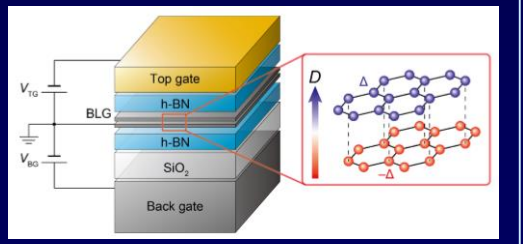
Łamiemy symetrię inwersji przestrzennej grafenu (dwuwarstwa grafenowa, pole elektryczne)

Dolinotronika *(Valleytronika)*

Dolinowy Efekt Halla







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